Norwegian University of Science and Technology
Department of Mathematical Sciences

MA1201 Linear Algebra and Geometry

Exercise set 02

Compulsory exercises

Hand in your solutions to these exercises. All answers must be justified.

Chapter 1.5

Exercise 1 Exercise 1 in chapter 1.5 of Elementary Linear Algebra.

(a) Yes, we add -5 times the first row to the second.

(b) No, this cannot be achieved with a single row operation. We need at least two.

(c) No, this matrix is not invertible.

(d) No, this cannot be achieved with a single row operation. We need at least two.

Exercise 2 Exercise 4 in chapter 1.5 of Elementary Linear Algebra.

(a) Add 3 times the first row to the second, which corresponds to

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

(b) Scale the third row by $\frac{1}{3}$, which corresponds to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

(c) Swap the first and fourth row, which corresponds to

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(d) Add $\frac{1}{7}$ times the third row to the first, which corresponds to

$$\begin{bmatrix} 1 & 0 & 1/7 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 3 Exercise 20a in chapter 1.5 of Elementary Linear Algebra. The inverse is

$$\begin{bmatrix} 0 & 0 & 0 & 1/k_4 \\ 0 & 0 & 1/k_3 & 0 \\ 0 & 1/k_2 & 0 & 0 \\ 1/k_1 & 0 & 0 & 0 \end{bmatrix}$$

Chapter 1.6

Exercise 4 Exercise 1 in chapter 1.6 of Elementary Linear Algebra.

The coefficient matrix is

$$A = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$$

Inverting this we get

$$A^{-1} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$$

Then the solution is

$$A^{-1}\mathbf{b} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Exercise 5 Exercise 14 in chapter 1.6 of Elementary Linear Algebra.

We set up the augmented matrix to solve the system.

$$\begin{bmatrix} 6 & -4 & b_1 \\ 3 & -2 & b_2 \end{bmatrix} \sim \begin{bmatrix} 6 & -4 & b_1 \\ 0 & 0 & b_2 - \frac{1}{2}b_1 \end{bmatrix}$$

From this we see that the only way for this system to be consistent is if $b_2 - \frac{1}{2}b_1 = 0$. In this case the solution is $x = \frac{1}{6}b_1 + \frac{2}{3}y$ with y a free variable.

Chapter 1.7

Exercise 6 Exercise 1 in chapter 1.7 of Elementary Linear Algebra.

a, and d are upper triangular. b is lower triangular, and c is diagonal.

a and c are invertible, since they have nonzero elements along the diagonal.

Exercise 7 Exercise 5 in chapter 1.7 of Elementary Linear Algebra.

Multiplication by a diagonal matrix on the left simply scales the rows.

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} -3 & 2 & 0 & 4 & -4 \\ 1 & -5 & 3 & 0 & 3 \\ -6 & 2 & 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} -15 & 10 & 0 & 20 & -20 \\ 2 & -10 & 6 & 0 & 6 \\ 18 & -6 & -6 & -6 & -6 \end{bmatrix}$$

Exercise 8 Exercise 47 in chapter 1.7 of Elementary Linear Algebra.

We are given that $A = A^T A$. Taking the transpose on both sides yields $A^T = (A^T A)^T = A^T (A^T)^T = A^T A = A$. Thus, $A = A^T$ which means that A is symmetric. Plugging this into the original equation we have $A = A^T A = AA = A^2$, so $A = A^2$. $\square$