Norwegian University of Science
and Technology
Department of Mathematical
Sciences

MA1201 Linear Algebra and Geometry

Exercise set 02

Compulsory exercises

Hand in your solutions to these exercises. All answers must be justified.

Chapter 1.3 - Matrix operations

Exercise 1 Exercise 1d, 1f, 2b, and 2c in chapter 1.3 of Elementary Linear Algebra.

1d is defined, AC gives us a 4×2-matrix, so $E(AC)$ gives us a 5×2-matrix.

1f is defined, $E(5B + A) = 5EB + EA$ and both EB and EA have size 5×5.

2b is not defined, D has rows of size 2, while C has columns of size 5, so we can not take their dot product.

2c is defined, both BC and $3D$ have size 4×2.

Exercise 2 Exercise 5a and 5b in chapter 1.3 of Elementary Linear Algebra.

$$AB = \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$$

BA is not defined since B has rows of size 2 while A has columns of size 3.

Chapter 1.4 - Inverses

Exercise 3 Exercise 10 in chapter 1.4 of Elementary Linear Algebra.

We set up the augmented matrix and rowreduce

$$\begin{bmatrix} \cos\theta & \sin\theta & 1 & 0 \\ -\sin\theta & \cos\theta & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta & \cos\theta & 0 \\ -\sin^2\theta & \sin\theta\cos\theta & 0 & \sin\theta \end{bmatrix} \sim$$
$$\begin{bmatrix} \cos^2\theta+\sin^2\theta & 0 & \cos\theta & -\sin\theta \\ -\sin^2\theta & \sin\theta\cos\theta & 0 & \sin\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cos\theta & -\sin\theta \\ -\sin^2\theta & \sin\theta\cos\theta & 0 & \sin\theta \end{bmatrix} \sim$$
$$\begin{bmatrix} 1 & 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta\cos\theta & \sin^2\theta\cos\theta & \sin\theta-\sin^3\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta\cos\theta & \sin^2\theta\cos\theta & \sin\theta(1-\sin^2\theta) \end{bmatrix} =$$
$$\begin{bmatrix} 1 & 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta\cos\theta & \sin^2\theta\cos\theta & \sin\theta\cos^2\theta \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \cos\theta & -\sin\theta \\ 0 & 1 & \sin\theta & \cos\theta \end{bmatrix}$$

So the inverse is

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

Exercise 4 Exercise 29 in chapter 1.4 of Elementary Linear Algebra.

$$A^2 - 9I = (A+3I)(A-3I) = \begin{bmatrix} 2 & 4 \\ 8 & -6 \end{bmatrix}$$

For Exercise 5 do at least one of 5a and 5b.

Exercise 5a Exercise 45a and 45b in chapter 1.4 of Elementary Linear Algebra.

45a

$$\begin{aligned} A(A^{-1}+B^{-1})B(A+B)^{-1} &= (AA^{-1}+AB^{-1})B(A+B)^{-1} \\ &= (I+AB^{-1})B(A+B)^{-1} \\ &= B(A+B)^{-1} + AB^{-1}B(A+B)^{-1} \\ &= B(A+B)^{-1} + A(A+B)^{-1} \\ &= (B+A)(A+B)^{-1} \\ &= (A+B)(A+B)^{-1} \\ &= I \end{aligned}$$

45b If we multiply the equation by A^{-1} on the left and A on the right we get

$$A^{-1}A(A^{-1}+B^{-1})B(A+B)^{-1}A = A^{-1}A == (A^{-1}+B^{-1})B(A+B)^{-1}A = I$$

Thus $(A^{-1}+B^{-1})$ is invertible with inverse $B(A+B)^{-1}A$.

Exercise 5b Exercise 46a in chapter 1.4 of Elementary Linear Algebra.

$$(I-A)^2 = (I-A)(I-A) = I^2 - AI - IA + A^2 = I - 2A + A = I - A$$