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Glossary

Engelsk	**Norsk**
norm	norm / lengde / størrelse
unit vector	enhetsvektor
acute angle	spiss vinkel
obtuse angle	stump vinkel

Compulsory exercises

Hand in your solutions to these exercises. All answers must be justified.

Chapter 3.1 - Vectors in $\mathbb{R}^n$

Exercise 1 Exercise 12c in chapter 3.1 of Elementary Linear Algebra.

$$\begin{aligned}(3\mathbf{u}-\mathbf{v})-(2\mathbf{u}+4\mathbf{w}) &= 3\mathbf{u}-2\mathbf{u}-\mathbf{v}-4\mathbf{w}\\ &= \mathbf{u}-\mathbf{v}-4\mathbf{w}\\ &= \begin{bmatrix}1\\2\\-3\\5\\0\end{bmatrix}-\begin{bmatrix}0\\4\\-1\\1\\2\end{bmatrix}-4\begin{bmatrix}7\\1\\-4\\-2\\3\end{bmatrix}=\end{aligned}$$

$$\begin{aligned}\begin{bmatrix}1\\2\\-3\\5\\0\end{bmatrix}-\begin{bmatrix}0\\4\\-1\\1\\2\end{bmatrix}-\begin{bmatrix}28\\4\\-16\\-8\\12\end{bmatrix} &= \begin{bmatrix}1-0-28\\2-4-4\\-3+1+16\\5-1+8\\0-2-12\end{bmatrix}\\ &= \begin{bmatrix}-27\\-6\\-14\\12\\-14\end{bmatrix}\end{aligned}$$

Exercise 2 Exercise 13 in chapter 3.1 of Elementary Linear Algebra.

$$\begin{aligned}3\mathbf{u}+\mathbf{v}-2\mathbf{w} &= 3\mathbf{x}+2\mathbf{w}\\ 3\mathbf{x} &= 3\mathbf{u}+\mathbf{v}-4\mathbf{w}\\ \mathbf{x} &= \mathbf{u}+\frac{1}{3}\mathbf{v}-\frac{4}{3}\mathbf{w}\\ &= \begin{bmatrix}-3\\2\\1\\0\end{bmatrix}+\begin{bmatrix}4/3\\7/3\\-1\\2/3\end{bmatrix}-\begin{bmatrix}20/3\\-8/3\\32/3\\4/3\end{bmatrix}\\ &= \begin{bmatrix}-3+4/3-20/3\\2+7/3+8/3\\1-1+32/3\\0+2/3-4/3\end{bmatrix}\\ &= \begin{bmatrix}-25/3\\7\\32/3\\-2/3\end{bmatrix}\end{aligned}$$

Exercise 3 Exercise 18 in chapter 3.1 of Elementary Linear Algebra.

The equation $a\mathbf{u} + b\mathbf{v} = (-8, 8, 3, -1, 7)$ corresponds to the system of equations

$$\begin{aligned} 2a - 2b &= -8 \\ a + 3b &= 8 \\ b &= 3 \\ a &= -1 \\ -a + 2b &= 7 \end{aligned}$$

We set up the augmented matrix and row reduce

$$\begin{pmatrix} 2 & -2 & -8 \\ 1 & 3 & 8 \\ 0 & 1 & 3 \\ 1 & 0 & -1 \\ -1 & 2 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -4 \\ 1 & 3 & 8 \\ 0 & 1 & 3 \\ 1 & 0 & -1 \\ -1 & 2 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -4 \\ 0 & 4 & 12 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Which gives the solution $a = -1$, $b = 3$.

Chapter 3.2 - Dot products

Exercise 4 Exercise 2a in chapter 3.2 of Elementary Linear Algebra.

The norm of $\mathbf{v}$ is $\|\mathbf{v}\| = \sqrt{1^1 + (-1)^2 + 2^2} = \sqrt{6}$. We have that $\frac{\mathbf{v}}{\|\mathbf{v}\|}$ is a unit vector pointing in the same direction as $\mathbf{v}$, so $-\frac{\mathbf{v}}{\|\mathbf{v}\|} = \begin{bmatrix} -1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{bmatrix}$ is a unit vector pointing in the opposite direction.

Exercise 5 Exercise 3a and 3b in chapter 3.2 of Elementary Linear Algebra.

$$\begin{aligned} \mathbf{u} + \mathbf{v} &= \begin{bmatrix} 2+1 \\ -2-3 \\ 3+4 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 7 \end{bmatrix} \\ \|\mathbf{u} + \mathbf{v}\| &= \sqrt{3^2 + (-5)^2 + 7^2} = \sqrt{83} \approx 9.1 \\ \|\mathbf{u}\| + \|\mathbf{v}\| &= \sqrt{2^2 + (-2)^2 + 3^2} + \sqrt{1^2 + (-3)^2 + 4^2} \\ &= \sqrt{17} + \sqrt{26} \approx 9.2 \end{aligned}$$

Exercise 6 Exercise 12a in chapter 3.2 of Elementary Linear Algebra.

The distance is

$$\|\mathbf{u} - \mathbf{v}\| = \sqrt{(1-5)^2 + (2-1)^2 + (-3-2)^2 + (-2)^2} = \sqrt{36} = 6$$

We also calculate

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= 1\cdot 5 + 2\cdot 2 + (-3)\cdot 2 + 0\cdot(-2) = 3\\ \|\mathbf{u}\| &= \sqrt{1^1 + 2^2 + (-3)^2} = \sqrt{14}\\ \|\mathbf{v}\| &= \sqrt{5^2 + 1^2 + (-2)^2 + 2^2} = \sqrt{34}\end{aligned}$$

The cosine of the angle is given as

$$\begin{aligned}\cos\theta &= \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\\ &= \frac{3}{\sqrt{14}\sqrt{34}} \approx 0.14\end{aligned}$$

Since this is positive, the angle is acute.

Exercise 7 Exercise 16 in chapter 3.2 of Elementary Linear Algebra. (Note: bold font symbols are vectors, normal font symbols are scalars)

16a makes sense. It is the product of two lengths.

16b does not make sense. The dot product gives us a scalar, and we cannot subtract a vector from a scalar.

16c makes sense. The dot product gives us a scalar, and we can subtract scalars from eachother.

16d makes sense. We are scaling **u**.

Exercise 8 Exercise 18a in chapter 3.2 of Elementary Linear Algebra.

We calculate

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= 4\cdot 1 + 1\cdot 2 + 1\cdot 3 = 9\\ \|\mathbf{u}\| &= \sqrt{4^2 + 1^2 + 1^2} = \sqrt{18}\\ \|\mathbf{v}\| &= \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}\end{aligned}$$

We see that $\mathbf{u}\cdot\mathbf{v} \leq \|\mathbf{u}\|\|\mathbf{v}\|$, because

$$9 \leq \sqrt{18}\cdot\sqrt{14} \approx 15.9.$$