22L

P ort : $P^T = P^{-1}$

Oppg 3 (a)

$$P = \begin{pmatrix} \bar{r}_1 \\ \hline \bar{r}_2 \\ \hline \bar{r}_3 \end{pmatrix} \implies P^T = \left(\bar{r}_1^T \mid \bar{r}_2^T \mid \bar{r}_3^T \right)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I = PP^T = \begin{pmatrix} \bar{r}_1 \\ \hline \bar{r}_2 \\ \hline \bar{r}_3 \end{pmatrix} \left(\bar{r}_1^T \mid \bar{r}_2^T \mid \bar{r}_3^T \right)$$

$$= \begin{pmatrix} \bar{r}_1 \bar{r}_1^T & \bar{r}_1 \bar{r}_2^T & \bar{r}_1 \bar{r}_3^T \\ \bar{r}_2 \bar{r}_1^T & r_2 r_2^T & r_2 r_3^T \\ r_3 r_1^T & r_3 r_2^T & r_3 r_3^T \end{pmatrix}$$

(Dropper strek over vektorene)

$$= \begin{pmatrix} r_1 \cdot r_1 & r_1 \cdot r_2 & r_1 \cdot r_3 \\ r_2 \cdot r_1 & r_2 \cdot r_2 & r_2 \cdot r_3 \\ r_3 \cdot r_1 & r_3 \cdot r_2 & r_3 \cdot r_3 \end{pmatrix}$$

$\implies r_1 \cdot r_1 = 1, \quad r_2 \cdot r_2 = 1, \quad r_3 \cdot r_3 = 1$

$r_1 \cdot r_2 = 0, \quad r_1 \cdot r_3 = 0, \quad r_2 \cdot r_3 = 0$

$\implies$ $||r_i|| = \sqrt{r_i \cdot r_i} = \sqrt{1} = 1$ og $r_1 \cdot r_2 = r_1 \cdot r_3 = r_2 \cdot r_3$

$\implies \{r_1, r_2, r_3\}$ ortonormal basis for $\mathbb{R}^3$

$\boxed{(AB)^T = B^T A^T}$

(b) $(P\bar{u})\cdot(P\bar{v}) = (P\bar{u})^T(P\bar{v}) =$

$$= \bar{u}^T \underbrace{P^T P}\, \bar{v}$$

$$= \bar{u}^T I \bar{v}$$

$$= \bar{u}^T \bar{v}$$

$$= \bar{u}\cdot\bar{v}$$

(c) $\|P\bar{v}\| = \sqrt{(P\bar{v})\cdot(P\bar{v})} \underset{\uparrow \text{ fra (b)}}{=} \sqrt{\bar{v}\cdot\bar{v}} = \|\bar{v}\|$

(d) Siden $PP^T = I$ får vi

$$1 = \det(I) = \det(PP^T) = \det(P)\cdot\det(P^T)$$

$$= \det(P)^2$$

siden generelt $\det(A^T) = \det(A)$.

Nå må $\det(P) \in \{1, -1\}$