MA1201 Linear Algebra and Geometry

Exercise set 06

Norwegian University of Science and Technology
Department of Mathematical Sciences

Compulsory exercises

Hand in your solutions to these exercises. All answers must be justified.

Chapter 3.5 - Cross product

Exercise 1 Do exercise 13 in chapter 3.5 of Elementary Linear Algebra.

The triangle is spanned by $A - C = (3, -2)$ and $B - C = (4, 2)$. The area of a triangle is half that of the associated parallellogram thus the area is

$$\left|\det\left(\begin{bmatrix} 3 & -2 \\ 4 & 2 \end{bmatrix}\right)\right| = 14$$

Exercise 2 Do exercise 21 in chapter 3.5 of Elementary Linear Algebra.

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \det\left(\begin{bmatrix} -2 & 0 & 6 \\ 1 & -3 & 1 \\ -5 & -1 & 1 \end{bmatrix}\right) = -92$$

Chapter 4.1 - Real vector spaces

Here are the vector space axioms as listed on page 203.

1. If $\mathbf{u}$ and $\mathbf{v}$ are in V, then $\mathbf{u} + \mathbf{v} \in V$.
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
3. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
4. There exists an object in V, called the zero vector, that is denoted by $\mathbf{0}$ and has the property that $\mathbf{0} + \mathbf{u} = \mathbf{u} + \mathbf{0} = \mathbf{u}$ for all $\mathbf{u}$ in V.
5. For each $\mathbf{u}$ in V, there is an object $-\mathbf{u}$ in V, called a negative of $\mathbf{u}$, such that $\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$.
6. If k is any scalar and $\mathbf{u} \in V$, then $k\mathbf{u} \in V$.

7. $k(\mathbf{u}+\mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
8. $(k+m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$
9. $k(m\mathbf{u}) = (km)(\mathbf{u})$
10. $1\mathbf{u} = \mathbf{u}$

Exercise 3 Do exercise 3-9 in chapter 4.1 of Elementary Linear Algebra.

3, 4, 6, and 9 are vector spaces. In exercise 5 axiom 5 and 6 fails. In exercise 7 axiom 8 fails. In exercise 8 axiom 1, 4, and 6 fails, and axiom 5 is illdefined without axiom 4.

Chapter 4.2 - Subspaces

Exercise 4 Do exercise 5 in chapter 4.2 of Elementary Linear Algebra.

(a) Let $p(x) = a_1x + a_2x^2 + a_3x^3$ and $q(x) = b_1x + b_2x^2 + b_3x^3$ be polynomials in P_3 with constant term 0, and let $k \in \mathbb{R}$ be a scalar. Then $p(x) + q(x) = (a_1 + b_1)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3$ and $kp(x) = ka_1x + ka_2x^2 + ka_3x^3$ also have constant term 0. Thus the set of such polynomials is a subspace.

(b) Let $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $q(x) = b_0 + b_1x + b_2x^2 + b_3x^3$ be such that

$$\sum_{i=0}^{3} a_i = 0 = \sum_{i=0}^{3} b_i,$$

and let $k \in \mathbb{R}$ be a scalar. Then $p(x) + q(x) = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3$ satisfies

$$\sum_{i=0}^{3} (a_i + b_i) = \sum_{i=0}^{3} a_i + \sum_{i=0}^{3} b_i = 0,$$

and $kp(x)$ satisfies

$$\sum_{i=0}^{3} ka_i = k \sum_{i=0}^{3} a_i = 0.$$

So the set of such polynomials is a subspace.