MA1201 Linear Algebra and Geometry

Exercise set 02

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Compulsory exercises

Hand in your solutions to these exercises. All answers must be justified.

Chapter 1.5

Exercise 1 Exercise 2 in chapter 1.5 of Elementary Linear Algebra.

(a) Yes, we have scaled the second row by $\sqrt{3}$.

(b) Yes, we have swapped row 1 and 3.

(c) Yes, we have added 9 times row 3 to row 2.

(d) No, this cannot be achieved with a single row operation. We need at least two.

Exercise 2 Exercise 3 in chapter 1.5 of Elementary Linear Algebra.

(a) Add 3 times the second row to the first, which corresponds to

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

(b) Scale the first row by $-\frac{1}{7}$, which corresponds to

$$\begin{bmatrix} -\frac{1}{7} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Add 5 times the first row to the third, which corresponds to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$$

(d) Swap the first and third row, which corresponds to

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 3 Exercise 19 in chapter 1.5 of Elementary Linear Algebra.

(a) The inverse is

$$\begin{bmatrix} 1/k_1 & 0 & 0 & 0 \\ 0 & 1/k_2 & 0 & 0 \\ 0 & 0 & 1/k_3 & 0 \\ 0 & 0 & 0 & 1/k_4 \end{bmatrix}$$

(b) The inverse is

$$\begin{bmatrix} 1/k & -1/k & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/k & -1/k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Chapter 1.6

Exercise 4 Exercise 1 in chapter 1.6 of Elementary Linear Algebra.

The coefficient matrix is

$$A = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$$

Inverting this we get

$$A^{-1} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$$

Then the solution is

$$A^{-1}\mathbf{b} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Exercise 5 Exercise 14 in chapter 1.6 of Elementary Linear Algebra.

We set up the augmented matrix to solve the system.

$$\begin{bmatrix} 6 & -4 & b_1 \\ 3 & -2 & b_2 \end{bmatrix} \sim \begin{bmatrix} 6 & -4 & b_1 \\ 0 & 0 & b_2 - \frac{1}{2}b_1 \end{bmatrix}$$

From this we see that the only way for this system to be consistent is if $b_2 - \frac{1}{2}b_1 = 0$. In this case the solution is $x = \frac{1}{6}b_1 + \frac{2}{3}y$ with y a free variable.

Chapter 1.7

Exercise 6 Exercise 2 in chapter 1.7 of Elementary Linear Algebra. a, c, and d are lower triangular; b and c are upper triangular; c is both, hence diagonal.

a and c are invertible because they have non-zero elements along the diagonal. b and d are not.

Exercise 7 Exercise 3 in chapter 1.7 of Elementary Linear Algebra.

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 4 & -1 \\ 4 & 10 \end{bmatrix}$$

Exercise 8 Exercise 4 in chapter 1.7 of Elementary Linear Algebra.

$$\begin{bmatrix} 1 & 2 & -5 \\ -3 & -1 & 0 \end{bmatrix} \begin{bmatrix} -4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 6 & -10 \\ 12 & -3 & 0 \end{bmatrix}$$

Exercise 9 Exercise 47 in chapter 1.7 of Elementary Linear Algebra.

We are given that $A = A^T A$. Taking the transpose on both sides yields $A^T = (A^T A)^T = A^T (A^T)^T = A^T A = A$. Thus, $A = A^T$ which means that A is symmetric. Plugging this into the original equation we have $A = A^T A = AA = A^2$, so $A = A^2$. $\square$