MA1103 Vector Calculus
Spring 2022

Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 8
Deadline: Mar. 13

You may write solutions in Norwegian or English, as preferable. Note that solutions should be written clearly and with justified arguments.

You can pose questions regarding homework or lecture etc. on the discussion forum Digital Mattelab, see https://wiki.math.ntnu.no/ma1103/2022v/start.

$\boxed{1}$ (14.1)

Consider the double integral

$$I = \iint_D (5 - x - y)\, dA,$$

where D is the rectangle $0 \leq x \leq 3$, $0 \leq y \leq 2$. P is the partition of D into six squares of side 1. Calculate the Riemann sums for I corresponding to the given choices of points (x^*_{ij}, y^*_{ij}) when:

a) (x^*_{ij}, y^*_{ij}) is the upper-left corner of each square

b) (x^*_{ij}, y^*_{ij}) is the centre of each square

$\boxed{2}$ (14.1)

Evaluate the double integrals below.

a) $\iint_R dA$, where R is the retangle $-1 \leq x \leq 3$, $-4 \leq y \leq 1$

b) $\iint_D (x+3)\, dA$, where D is the half-disk $0 \leq y \leq \sqrt{4 - x^2}$

$\boxed{3}$ (14.2)

Evaluate the double integrals below by iteration.

a) $\iint_R (x^2 + y^2)\, dA$, where R is the rectangle $0 \leq x \leq a$, $0 \leq y \leq b$

b) $\iint_S \Big(\sin(x) + \cos(y)\Big)\, dA$, where S is the square $0 \leq x \leq \frac{\pi}{2}$, $0 \leq y \leq \frac{\pi}{2}$

c) $\iint_D \ln(x)\, dA$, where D is the finite region in the first quadrant bounded by the line $2x + 2y = 5$ and the hyperbola $xy = 1$

$\boxed{4}$ (13.2), (13.3)

a) Find the maximum and minimum values of $xy^2 + yz^2$ over the ball $x^2 + y^2 + z^2 \leq 1$.

b) Find the maximum and minimum values of $f(x, y, z) = 4 - z$ on the ellipse formed by the intersection of the cylinder $x^2 + y^2 = 8$ and the plane $x + y + z = 1$.

$\boxed{5}$ (14.3)

Determine whether the given integrals below converge or diverge. Evaluate those that converge.

a) $\iint_Q e^{-x-y}\, dA$, where Q is the first quadrant of the xy-plane

b) $\iint_S \frac{y}{1+x^2}\, dA$, where S is the strip $0 < y < 1$ in the xy-plane

$\boxed{6}$ (implicit function theorem)

Show that the equations

$$\begin{cases} xe^y + uz - \cos(v) = 2 \\ u\cos(y) + x^2 v - yz^2 = 1 \end{cases}$$

can be solved for u and v as functions of x, y, and z near the point P_0 where $(x, y, z) = (2, 0, 1)$ and $(u, v) = (1, 0)$, and find $\frac{\partial u}{\partial z}$ at $(x, y, z) = (2, 0, 1)$.

Hint: Check the conditions before using the implicit function theorem.