MA1103 Vector Calculus
Spring 2022

Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 7
Deadline: Mar. 06

You may write solutions in Norwegian or English, as preferable. Note that solutions should be written clearly and with justified arguments.

You can pose questions regarding homework or lecture etc. on the discussion forum Digital Mattelab, see https://wiki.math.ntnu.no/ma1103/2022v/start.

1 (13.1)

Find and classify the critical points of the given functions below.

a) $f(x, y) = xy - x + y$

b) $f(x, y) = \cos(x) + \cos(y)$

c) $f(x, y) = xe^{-x^3+y^3}$

2 (13.2)

Determine if there exist global maximal and minimal values for the following functions, and find them when possible.

a) $f(x, y) = xy - x^3y^2$, over the square $0 \leq x \leq 1$, $0 \leq y \leq 1$

b) $f(x, y) = \frac{x-y}{1+x^2+y^2}$, on the upper half-plane $y \geq 0$

c) $f(x, y) = x + 8y + \frac{1}{xy}$, in the first quadrant $x > 0$, $y > 0$

3 (13.3)

Use the method of Lagrange multipliers to maximize x^3y^5 subject to the constraint $x + y = 8$.

4 (13.3)

Find the maximum and minimum values of the function $f(x, y, z) = x$ over the curve of intersection of the plane $z = x + y$ and the ellipsoid $x^2 + 2y^2 + 2z^2 = 8$.

5 (13.3)

A space probe shaped as an ellipsoid $4x^2 + y^2 + 4z^2 \leq 16$ enters Earth's atmosphere, and its surface begins to heat up. One hour later, the temperature on the surface of the space probe at point (x, y, z) is given by $T = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the surface of this probe.

$\boxed{6}$ (inverse function theorem)

Consider the following system of equations

$$\begin{cases} u = x^2 - xy, \\ v = y - 2x. \end{cases}$$

Show that near (x_0, y_0) (where $y_0 \neq 0$), (x, y) can be written as differentiable function of (u, v). Compute $\frac{\partial y}{\partial v}(u_0, v_0)$.

Hint: Check the conditions before using the inverse function theorem.

$\boxed{7}$ (implicit function theorem)

Give $F(x, y, z) = C$, where $C \in \mathbb{R}$ is a constant. Let $\mathbf{p}_0 = (x_0, y_0, z_0)$ and $F(\mathbf{p}_0) = C$. Assume $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}$ are continuous and not equal to 0 at $\mathbf{p}_0$.

a) Show that near $\mathbf{p}_0$, we can write

$$x = f_1(y, z), \quad y = f_2(x, z), \quad z = f_3(x, y),$$

where f_1, f_2, f_3 are differentiable functions on their variables.

b) Prove that

$$\frac{\partial f_1}{\partial y}(\mathbf{p}_0)\frac{\partial f_2}{\partial z}(\mathbf{p}_0)\frac{\partial f_3}{\partial x}(\mathbf{p}_0) = -1.$$

Hint: Check the conditions before using the implicit function theorem.