Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 3
Deadline: Feb. 06

You may write solutions in Norwegian or English, as preferable. Note that solutions should be written clearly and with justified arguments.

You can pose questions regarding homework or lecture etc. on the discussion forum Digital Mattelab, see https://wiki.math.ntnu.no/ma1103/2022v/start.

1 Compute $\det(A+B)$ and $\det(AB)$ for

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

2 For what values of the parameter λ is the length $s(T)$ of the curve $\mathbf{r} = (t, \lambda t^2, t^3)$ $(0 \le t \le T)$ given by $s(T) = T + T^3$?

3 Find the Frenet frame $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ for the curve (1) at the point indicated.

$$\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k} \qquad \text{at} \quad (1,1,1), \tag{1}$$

where $\mathbf{i} = (1,0,0), \mathbf{j} = (0,1,0), \mathbf{k} = (0,0,1)$.

4 Let $\mathbf{u}, \mathbf{v}$ be two given vectors in a (real) vector space V equipped with an inner product $\langle , \rangle_V$, and let $S(\mathbf{v})$ denote the (real) linear span of $\mathbf{v}$. The **projection of u on v**, $P_{\mathbf{v}}(\mathbf{u})$, is the point in $S(\mathbf{v})$ closest to $\mathbf{u}$.

a) Determine a formula for $P_{\mathbf{v}}(\mathbf{u})$.

b) Find $P_{(1,0,-2)}(1,2,3)$.

5 Specify the natural domain of the function below.

$$f\colon (x,y) \mapsto \arcsin(x+y)$$

6 Determine in each case below whether the limit exists, and in the case it does, give its value.

a) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{y}$

b) $\lim_{(x,y)\to(1,\pi)} \frac{\cos(xy)}{1-x-\cos(y)}$

c) $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{2x^4+y^4}$

7 How can the function

$$f(x,y) = \frac{x^3-y^3}{x-y}, \qquad (x \neq y),$$

be defined along the line $x = y$ so that the resulting function is continuous on the whole xy-plane?

8 Find all first order partial derivatives of the given functions below.

a) $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$

b) $f(x,y,z) = x^3y^4z^5$

c) $f(x,y,z) = \ln(1+e^{xyz})$

9 Define a function

$$f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(1) Calculate $f_x(x,y)$ and $f_y(x,y)$ at all points (x,y) in the plane.

(2) Is f continuous at $(0,0)$? Are f_x and f_y continuous at $(0,0)$?