MA1103 Exercise Set 1

Norwegian University of Science and Technology

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IMPORTANT: The deadline for this exercise is Friday Jan. 22. You may write solutions in Norwegian or English, as preferable. You may cooperate, but should be able to explain your solutions and reasoning in short oral presentations.

Problem 1

Identify the type of curve and specify the asymptotes satisfying the given equation below (1). Sketch the set of points in the plane for it.

$$4x^2 - y^2 - 4y = 0. \tag{1}$$

Problem 2

a)

Give the equation satisfied by a general point $(x, y) \in S$ where S denotes the circle with center $(1, -\pi)$ and radius 2. Give a parametrization of S of the form $\mathbf{r} = \mathbf{r}(t) = (x(t), y(t)),\ t \in [0, 2\pi]$.

b)

Give the equation satisfied by a general point (x, y) on the ellipse centred at $(e, -\sqrt{2})$ with semi-major axis equal to $a = \sqrt{3}$ and semi-minor axis equal to $b = \cos\frac{\pi}{4}$. Give a parametrization of the ellipse of the form $\mathbf{r} = \mathbf{r}(t) = (x(t), y(t)),\ t \in [0, 2\pi]$.

c)

Sketch the circle S and the ellipse in b).

Problem $\boxed{3}$

Where, if anywhere, does the curve below (2) fail to be smooth in the sense of A&E [1]?

$$x = t^3, \quad y = t - \sin t. \tag{2}$$

Problem $\boxed{4}$

a)

Consider the function f given by $f(x) = (x-1)(x+1)$ for $x \in [-1, 1]$. Determine the length of the graph of f.

b)

Consider the polar curve $r = f(\theta)$ where $f(\theta) = \theta$. Sketch the curve for $\theta \in [0, 3\pi]$ and determine its arclength over this interval. What kind of curve is this?

Problem $\boxed{5}$

Transform the given polar equation below (3) to rectangular coordinates, and identify which type of curve it represents.

$$r = \frac{2}{1 + \sin\theta}. \tag{3}$$

Problem $\boxed{6}$

Consider the polar curve $r = f(\theta)$ given by $f(\theta) = \sin(2\theta),\ \theta \in [0, 2\pi]$. This curve bounds a finite region in the plane. Determine the area of the this region.

Problem $\boxed{7}$

One leaf of the lemniscate $r^2 = \cos 2\theta$ is rotated (a) about the x-axis and (b) about the y-axis. Find the area of the surface generated in each case.

Problem $\boxed{8}$

For each $n \in \mathbb{N}$ consider the polar curve $r_n = f(\theta; n)$ where $f(\theta; n) = \sin(n\theta),\ \theta \in [0, 2\pi]$. The curve r_n bounds a finite region R_n in the plane. Determine the area of R_n for general $n \in \mathbb{N}$. **Hint:** consider even

and odd n separately.

References

[1] R.A. Adams, C. Essex. *Calculus, A Complete Course*, Ninth edition, Pearson.