

## Formler og konvensjoner

**Diskriminanten i andrederiverttesten:**

$$\Delta = AC - B^2 \quad \text{der} \quad A = f_{xx}, B = f_{xy}, C = f_{yy}$$

### Implisitt og Omvendt Funksjonsteoremet

Omvendt Funksjonsteoremet  $\mathbf{F} : U \subset \mathbf{R}^m \rightarrow \mathbf{R}^m$ :

La  $\mathbf{F}$  være kontinuerlig deriverbar og la  $U$  være en omegn av  $\bar{\mathbf{x}}$ . Anta at  $\mathbf{F}'(\bar{\mathbf{x}})$  er inverterbar. Da finnes en omegn  $\bar{\mathbf{x}} \in U_0 \subset U$  slik at  $\mathbf{F}$  er injektiv. Den omvendte funksjonen  $\mathbf{G} : V = \mathbf{F}(U_0) \rightarrow U_0$  er deriverbar i  $\bar{\mathbf{y}}$  med Jacobi-matrise  $\mathbf{G}'(\bar{\mathbf{y}}) = \mathbf{F}'(\bar{\mathbf{x}})^{-1}$ .

Implisitt Funksjonsteoremet  $\mathbf{F} : U \subset \mathbf{R}^{m+k} \rightarrow \mathbf{R}^k$ :

La  $\mathbf{F}$  være kontinuerlig deriverbar og la  $U$  være en åpen mengde. Anta at  $(\bar{\mathbf{x}}, \bar{\mathbf{y}}) \in U$  og  $\mathbf{F}(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = 0$ . Anta videre at  $k \times k$ -matrisen  $\frac{\partial \mathbf{F}}{\partial \mathbf{y}}(\bar{\mathbf{x}}, \bar{\mathbf{y}})$  er inverterbar. Da finnes det en omegn  $\bar{\mathbf{x}} \in U_0$  slik at for en hver  $\mathbf{x} \in U_0$  finnes det en unik vektor  $\mathbf{G}(\mathbf{x})$  slik at  $\mathbf{F}(\mathbf{x}, \mathbf{G}(\mathbf{x})) = 0$ . Funksjonen  $\mathbf{G} : U_0 \rightarrow \mathbf{R}^k$  er deriverbar og  $\mathbf{G}'(\mathbf{x}) = -\left(\frac{\partial \mathbf{F}}{\partial \mathbf{y}}(\mathbf{x}, \mathbf{G}(\mathbf{x}))\right)^{-1}\left(\frac{\partial \mathbf{F}}{\partial \mathbf{x}}(\mathbf{x}, \mathbf{G}(\mathbf{x}))\right)$ .

### Formler for Skifte av Variabler:

$$dx dy = \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv, \quad dx dy dz = \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| du dv dw$$

Sylinderkoordinater  $(r, \theta, z)$ :

$$\begin{aligned} x &= r \cos \theta, & y &= r \sin(\theta), & z &= z, \\ r^2 &= x^2 + y^2, & dx dy dz &= r dr d\theta dz \end{aligned}$$

Kulekoordinater  $(\rho, \varphi, \theta)$ :

$$\begin{aligned} x &= \rho \cos(\theta) \sin(\varphi), & y &= \rho \sin(\theta) \sin(\varphi), & z &= \rho \cos(\varphi), \\ \rho^2 &= x^2 + y^2 + z^2, & dx dy dz &= \rho^2 \sin(\varphi) d\rho d\theta d\varphi \end{aligned}$$

**Flateintegral:**

$$dS = |\mathbf{r}_u \times \mathbf{r}_v| du dv$$

Spesialtilfellet  $z = g(x, y)$ :

$$dS = \sqrt{1 + g_x^2 + g_y^2} dx dy$$

**Vektoranalyse**  $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ :

$$\text{curl}(\mathbf{F}) = \left( \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right), \quad \text{div}(\mathbf{F}) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\text{Green sitt teorem: } \int_{\partial D} P dx + Q dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

$$\text{Stokes sitt teorem: } \int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

$$\text{Divergensteoremet: } \iint_{\partial V} \mathbf{F} \cdot d\mathbf{S} = \iint_{\partial V} \mathbf{F} \cdot \mathbf{n} dS = \iiint_V \text{div}(\mathbf{F}) dV$$