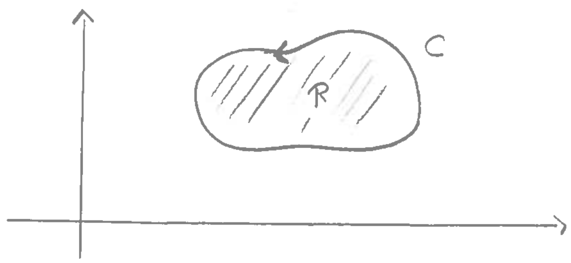
REPETITION 20/03



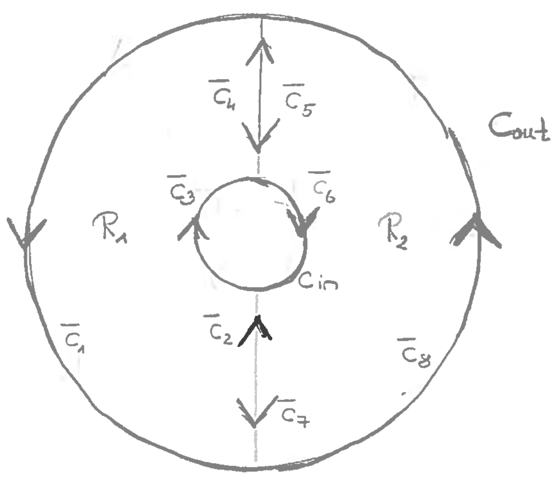
$F : R \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ $\quad C^1$ vector field

$$\boxed{\iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, d(x,y) = \int_C F \cdot dr \qquad \underline{\text{Greens theorem}}}$$

- Let $F(x,y) = (0, x)$ or $F(x,y) = -(y, 0)$ or $F(x,y) = \frac{1}{2}(-y, x)$

$$\Rightarrow \quad \text{area}(A) = \iint_R 1 \, d(x,y) = \int_C F \, dr$$

- Greens theorem applies to regions with „holes“



$C = C_{out} + C_{in}$

$$\iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, d(x,y) = \left(\iint_{R_1} + \iint_{R_2} \right) \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, d(x,y)$$

$$\underset{\text{in } R_1, R_2}{\overset{\text{Green}}{=}} \left(\int_{\overline{C}_1} + \int_{\overline{C}_2} + \int_{\overline{C}_3} + \int_{\overline{C}_4} + \underbrace{\int_{\overline{C}_5}}_{= -\int_{\overline{C}_4}} + \int_{\overline{C}_6} + \underbrace{\int_{\overline{C}_7}}_{= -\int_{\overline{C}_2}} + \int_{\overline{C}_8} \right) F \cdot dr$$

$$= \int_C F \, dr$$