

REPETITION 15/01

\bar{f} is continuous in $a \iff \lim_{x \rightarrow a} \bar{f}(x) = \bar{f}(a)$

a/

$$f(x, y) = \begin{cases} \frac{\sin(x^2 + y^2)}{x^2 + y^2} & (x, y) \neq 0 \\ 1 & (x, y) = 0 \end{cases}$$

$$\lim_{(x, y) \rightarrow (0, 0)} f(x, y) = \lim_{(x, y) \rightarrow (0, 0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2} = \lim_{r \rightarrow 0} \frac{\sin(r)}{r} = 1 = f(0, 0)$$

$\Rightarrow f$ is cont in $(0, 0)$

b/ $f(x, y) = \frac{x^2}{x^2 + y^2}$ is defined on $\mathbb{R}^2 \setminus \{(0, 0)\}$

$\rightarrow f$ has no cont. extension on \mathbb{R}^2

$$\lim_{(x, 0) \rightarrow (0, 0)} f(x, 0) = \lim_{x \rightarrow 0} \frac{x^2}{x^2} = 1$$

$$\lim_{(0, y) \rightarrow (0, 0)} f(0, y) = \lim_{y \rightarrow 0} \frac{0}{0 + y^2} = 0$$

