REPETITION 14/03

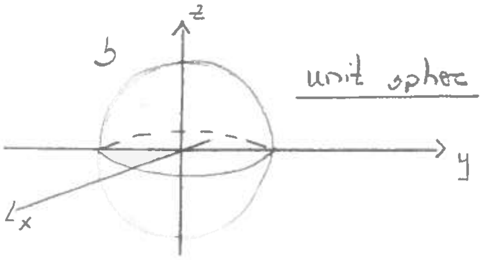
Let S be a surface parameterized by

$$r : A \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$

and $f : S \subset \mathbb{R}^3 \longrightarrow \mathbb{R}$, then

$$\iint_S f \, dS := \iint_A f(r(u,v)) \left\| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right\| d(u,v)$$

is the <u>surface integral of f over S</u>



<u>unit sphere</u>

Parameterization

$$r : [0, 2\pi] \times [0, \pi] \longrightarrow \mathbb{R}^3$$

$$r(\theta, \varphi) = (\cos\theta \sin\varphi, \sin\theta \sin\varphi, \cos\varphi)$$

$$\Rightarrow \quad \frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial \varphi} = -\sin\varphi \cdot r(\theta, \varphi)$$

$$\Rightarrow \quad \left\| \frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial \varphi} \right\| = \sin\varphi$$

If $f(x,y,z) = z^2$, then

$$\iint_S f \, dS = \int_0^{2\pi} \int_0^{\pi} \cos^2\varphi \cdot \sin\varphi \, d\varphi \, d\theta = \frac{4}{3}\pi$$