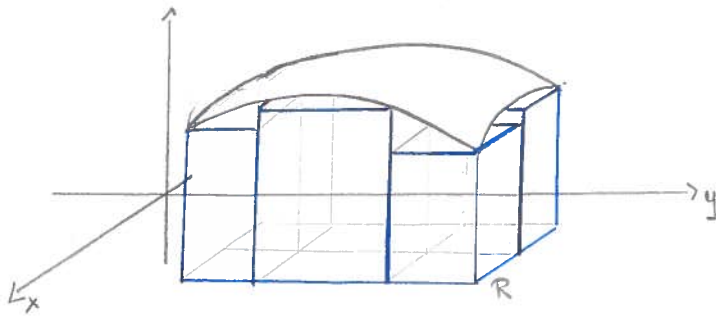


REPETITION 03/03



Π partition of R

f is (Riemann) integrable \Leftrightarrow

$$\underbrace{\int\int_{\bar{R}} f(x,y) d(x,y)}_{\text{lower Riemann integral}} = \underbrace{\int\int_{\bar{R}} f(x,y) d(x,y)}_{\text{upper Riemann integral}} =: \int\int_{\bar{R}} f(x,y) d(x,y)$$

Theorem Let $R = [a,b] \times [c,d]$ and $f: R \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ be integrable

If $y \mapsto f(x,y)$ is integrable on $[c,d]$

$$\Rightarrow \int\int_{\bar{R}} f(x,y) d(x,y) = \int_a^b \int_c^d f(x,y) dy dx$$

If $x \mapsto f(x,y)$ is integrable on $[a,b]$

$$\Rightarrow \int\int_{\bar{R}} f(x,y) d(x,y) = \int_c^d \int_a^b f(x,y) dx dy$$