LØSNINGS-SKISSER Øving 3 MA1103

Meld fra om feil! KK

Oppg 1

a) $\lim_{(x,y)\to(1,0)} \frac{e^{x+y}}{x^2+3y} = \frac{e^{1+0}}{1^2+3\cdot 0} = \underline{\underline{e}}$

b) $\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy}\cos(x+y) = 1\cdot\cos(0+0) = \underline{\underline{1}}$

Hint i Boka

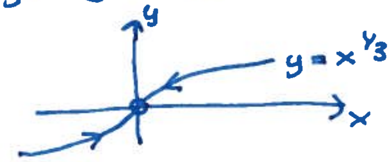
Oppg 2

Skal vise at $f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^6} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$

ikke er kont. i $(0,0)$.

Langs aksene går f mot 0. Prøver med $x^2 = y^6$ eller $x = y^3$ ($y = x^{1/3}$):

$f(y^3, y) = \frac{y^3\cdot y^3}{y^6+y^6} = \frac{1}{2}$ når $y \neq 0$: $\lim_{y\to 0} f(y^3,y) = \frac{1}{2} \neq f(0,0)$.



Oppg. 3

a) $f(x,y) = x^3y + 3xy^4$; $\underline{f_x = 3x^2y + 3y^4,\ f_y = x^3 + 12xy^3}$

b) $f(x,y) = \frac{x^2+x^3}{y}$; $\underline{f_x = \frac{2x+3x^2}{y},\ f_y = -\frac{x^2+x^3}{y^2}}$

c) $f(x,y) = x^2\ln(xy^2)$; $\underline{f_x} = 2x\ln(xy^2) + x^2\frac{y^2}{xy^2}$

$= \underline{2x\ln(xy^2) + x}$

$\underline{f_y} = x^2\frac{2xy}{xy^2} = \underline{\frac{2x^2}{y}}$

d) $f(x,y,z) = (x+y)e^{-z}$; $\underline{f_x = e^{-z},\ f_y = e^{-z},\ f_z = -(x+y)e^{-z}}$

Oppg. 4

a) $f(x,y) = x^2y$; $\underline{\text{grad } f = (2xy, x^2)}$

b) $f(u,v,w) = w e^{u\cos v}$

$\underline{\text{grad } f(u,v,w) = (w\cos v\, e^{u\cos v}, -w\sin v\, e^{u\cos v}, e^{u\cos v})}$

Oppg. 5

De partielle deriverte er kont. i a, og f dermed deriverbar i a (T 2.4.10). Vi har da (S 2.4.8)

$$f'(a;r) = \nabla f(a)\cdot r$$

a) $\nabla f(a) = (3y, 3x+2y)_{(1,2)} = \underline{(6,7)}$

$\nabla f(a)\cdot(3,-1) = (6,7)\cdot(3,-1) = \underline{\underline{11}}$

b) $\nabla f(a) = \left(\frac{1}{x+y^2}, \frac{2y}{x+y^2}\right)_{(1,0)} = \underline{(1,0)}$

$\nabla f(a)\cdot(-1,1) = (1,0)\cdot(-1,1) = \underline{\underline{-1}}$

Oppg. 6

Funksjonen vokser hurtigst i retning gradienten (f deriverbar i a som over). Altså:

a) $\nabla f(a) = (-2xy, -x^2+21y^2)_{(4,-3)} = \underline{\underline{(24, 173)}}$

b) $\nabla f(a) = (2x\,e^z, -2y\,e^z, (x^2-y^2)e^z)_{(1,-1,3)} = \underline{\underline{2e^3(1,1,0)}}$

Oppgave 7

$$f(x,y) = \begin{cases} \dfrac{x^2y}{x^4+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 \end{cases}$$

a) $\dfrac{\partial f}{\partial x}(0,0) = \lim\limits_{h\to 0} \dfrac{f(h,0)-f(0,0)}{h} = \lim\limits_{h\to 0} \dfrac{0-0}{h} = \underline{\underline{0}}$

$\dfrac{\partial f}{\partial y}(0,0) = \lim\limits_{h\to 0} \dfrac{f(0,h)-f(0,0)}{h} = \lim\limits_{h\to 0} \dfrac{0-0}{h} = \underline{\underline{0}}$

$\underline{\underline{\nabla f(0,0) = (0,0)}}$ nullvektor

b) (De retningsderiverte eksisterer, se c))

f er <u>ikke</u> kontinuerlig i $(0,0)$:

$f(x,x^2) \underset{x\neq 0}{=} \dfrac{x^2 \cdot x^2}{x^4+(x^2)^2} = \dfrac{1}{2}$

$y = x^2$ $(x \neq 0)$, $f(x,y) = \frac{1}{2}$

$\lim\limits_{x\to 0} f(x,x^2) = \frac{1}{2} \neq f(0,0)$. Da heller <u>ikke</u> deriverbar (S 2.4.13)!

c) $r = (r_1, r_2)$, $r_2 \neq 0$

$f'(0;r) \underset{\text{DEF}}{=} \lim\limits_{h\to 0} \dfrac{f(hr_1, hr_2) - f(0,0)}{h} = \lim\limits_{h\to 0} \dfrac{h^2 r_1^2 h r_2}{(h^4 r_1^4 + h^2 r_2^2)h}$

$= \lim\limits_{h\to 0} \dfrac{r_1^2 r_2}{h^2 r_1^4 + r_2^2} = \underline{\underline{\dfrac{r_1^2}{r_2}}}$

d)

$f'(0,r) = \underline{r_1^2/r_2}$, $\nabla f(0,0) \cdot (r_1, r_2) = (0,0) \cdot (r_1 r_2) = \underline{0}$

Ikke i strid med S 2.4.8 som forutsetter f deriverbar i a.

Oppg. 8

Underforstår $f(0,0) = 0$. T 2.4.10 nærliggende.

$f_x = 3(x+y)^2 \underbrace{\sin\left(\tfrac{1}{x+y}\right)} - (x+y)\underbrace{\cos\left(\tfrac{1}{x+y}\right)}$ $\rightarrow 0$ når $(x,y) \to (0,0)$

f_y = samme $\qquad |\sim| \leq 1$

Part. der. kont. i $(0,0)$