

Vector Calculus

1 The geometry of Euclidean spaces

- a) Length, distances
- b) Dot product, cross product, determinants
- c) Cylindrical and spherical coordinates

2 Differentiation

- a) Level sets
- b) Open sets, limit, continuity ($\varepsilon - \delta$ -definition)
- c) Partial derivatives and properties
- d) Definition of differentiability
- e) The gradient and the differential of a function, chain rule
- f) Curves: parametrization, length, speed, tangent vector, tangent line
- g) Directional derivatives

3 Higher-order derivatives: maxima and minima

- a) Iterated partial derivatives and properties
- b) Taylor polynomials
- c) Critical points, second derivative test for maxima and minima
- d) Constrained maxima (Lagrange-Method)

4 Vector valued functions

- a) Newton's Second Law $F(c(t)) = ma(t)$
- b) Vector fields, flow lines, divergence and curl

5 Double and triple integrals

- a) Concept of Riemann sums
- b) Definition of double and triple integrals and when do they exist?
- c) Changing the order of integration and when is it allowed?
- d) Properties of the integral
- e) x - and y -simple regions and integration over simple regions.
- f) Mean value Theorem for integrals

6 The change of variables formula

- a) Injective, surjective, bijective map
- b) Change of variables formula
- c) Average and center of mass

7 Integrals over paths and surfaces

- a) The path integral ($f : C \rightarrow \mathbb{R}$)
- b) The line integral ($\mathbf{F} : C \rightarrow \mathbb{R}^n$), line integral of gradient vector fields (generalization of the fundamental theorem of calculus)
- c) Dependence on the orientation of the curve?
- d) Parametrization of surfaces, regular surfaces, special case: the surface is given by a graph of a function
- e) The integral of a scalar valued function over a surface ($f : S \rightarrow \mathbb{R}$), special case: the area of a surface
- f) The integral of a vector field over a surface ($\mathbf{F} : S \subset \mathbb{R}^3 \rightarrow \mathbb{R}^3$), physical interpretation.
- g) Dependence on the orientation of the surface?

8 The integral theorems of vector analysis

- a) Conditions and formulation of the theorems of Green, Stokes and Gauss
- b) Conservative fields and their characterization.