Exercises

Vector Calculus (MA1103)

Exercise 9

Exercise 1 (5.3: 3)

Evaluate the following iterated integrals and draw the regions D determined by the limits. State whether the regions are x–simple, y–simple, or both.

a) $\int_0^1 \int_0^{x^2} dy\, dx$

b) $\int_1^2 \int_{2x}^{3x+1} dy\, dx$

c) $\int_0^1 \int_1^{e^x} (x+y) dy\, dx$

d) $\int_0^1 \int_{x^3}^{x^2} y\, dy\, dx$

Exercise 2 (5.3: 6)

Using double integrals, determine the area of an ellipse with semiaxis of length a and b.

Exercise 3 (5.3: 12)

Evaluate the following double integral:

$$\iint_D \cos(y)\, dx\, dy,$$

where the region D is bounded by $y = 2x, y = x, x = \pi$, and $x = 2\pi$.

Exercise 4 (5.4: 5)

Change the order of integration and evaluate:

$$\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx\, dy.$$

Exercise 5 (5.4: 7)

If $f(x,y) = e^{\sin(x^2+y^2)}$ and $D = [-\pi, \pi] \times [-\pi, \pi]$, show that

$$\frac{1}{e} \le \iint_D f(x,y)\, dA \le e.$$

Exercise 6 (5.5: 2)

Evaluate the following triple integral

$$\iiint_W \sin(x)\, dx\, dy\, dz,$$

where W is the solid given by $0 \le x \le \pi, 0 \le y \le 1$, and $0 \le z \le x$.

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Exercise 7 (5.5: 30)

Let W be the region bounded by the planes $x = 0, y = 0, z = 0, x + y = 1$, and $z = x + y$.

a) Find the volume of W

b) Evaluate $\iiint_W x\, dx\, dy\, dz$

c) Evaluate $\iiint_W y\, dx\, dy\, dz$

Exercise 8 (**A** 5.5: 31)

Let f be continuous and let B_ε be the ball of radius ε centered at the point (x_0, y_0, z_0). Let $\mathrm{vol}(B_\varepsilon)$ be the volume of B_ε. Prove that

$$\lim_{\varepsilon \to 0} \frac{1}{\mathrm{vol}(B_\varepsilon)} \iiint_{B_\varepsilon} f(x, y, z)\, dV = f(x_0, y_0, z_0).$$

Exercise 9 (6.2: 3)

Let D be the unit disk: $x^2 + y^2 \leq 1$. Evaluate

$$\iint_D e^{x^2+y^2}\, dx\, dy$$

by making a change of variables to polar coordinates.

Exercise 10 (6.2: 5)

Let $T(u, v) = (x(u, v), y(u, v))$ be the mapping defined by $T(u, v) = (4u, 2u+3v)$. Let D^* be the rectangle $[0, 1] \times [1, 2]$. Find $D = T(D^*)$ and evaluate

a) $\iint_D xy\, dx\, dy$

b) $\iint_D (x - y)\, dx\, dy$

A: This exercise is more theoretical (and might therefore be more difficult).

The exercise can be also found (under the given number in brackets) in the book *Vector Calculus* by J. E. Marsden and A. Tromba.