Exercises
Vector Calculus (MA1103)
Exercise 8

Exercise 1 (Proof of Theorem 4.2)

Show that if $\mathbf{F} \in C^2(\mathbb{R}^n, \mathbb{R}^n)$ is a vector field, then $\operatorname{div} \operatorname{curl} \mathbf{F} = 0$.

Exercise 2 (4.4: 8)

Sketch a few flow lines for $\mathbf{F}(x, y) = (-3x, -y)$. Calculate $\operatorname{div} \mathbf{F}$ and explain why your answer is consistent with your sketch.

Exercise 3 (4.4: 24)

Suppose that $f : \mathbb{R}^3 \to \mathbb{R}$ is a C^2 function (twice continuously differentiable). Which of the following expressions are meaningful, and which are nonsense? For those which are meaningful, decide whether the expression defines a scalar (that is real-valued) function or a vector field.

a) $\operatorname{curl}(\nabla f)$

b) $\nabla(\operatorname{curl} f)$

c) $\operatorname{div}(\nabla f)$

d) $\nabla(\operatorname{div} f)$

e) $\operatorname{curl}(\operatorname{div} f)$

f) $\operatorname{div}(\operatorname{curl} f)$

Exercise 4 (4.4: 26)

Suppose that $f, g, h : \mathbb{R} \to \mathbb{R}$ are differentiable. Show that the vector field $\mathbf{F}(x, y, z) = (f(x), g(y), h(z))$ is irrotational.

Please turn the page

Exercise 5 (4.4: 37)

Let $\mathbf{F}(x,y,z) = (2xz^2, 1, y^3zx)$ and $f(x,y,z) = x^2y$. Compute the following quantities

a) ∇f

b) $\operatorname{curl}\mathbf{F}$

c) $\mathbf{F} \times (\nabla f)$

d) $\mathbf{F} \cdot (\nabla f)$

Exercise 6 (5.2: 2)

Evaluate each of the following integrals if $R = [0,1] \times [0,1]$.

a) $\iint_R x^m y^n \, dx \, dy$, where $m, n > 0$

b) $\iint_R (ax + by + c) \, dx \, dy$

c) $\iint_R \sin(x+y) \, dx \, dy$

Exercise 7 (5.2: 9)

Let f be continuous on $[a,b]$ and g continuous on $[c,d]$. Show that

$$\iint_R [f(x)g(y)] \, dx \, dy = \left[\int_a^b f(x) \, dx\right] \left[\int_c^d g(y) \, dy\right],$$

where $R = [a,b] \times [c,d]$.

Exercise 8 (5.2: 11)

Compute the volume of the solid bounded by the graph $z = x^2 + y$, the rectangle $R = [0,1] \times [1,2]$ and the *vertical sides* of R.

Exercise 9 (**A** 5.2: 18)

Let f be continuous, $f \geq 0$, on the rectangle R. If $\iint_R f \, dA = 0$, prove that $f = 0$ on R.

A: This exercise is more theoretical (and might therefore be more difficult).

The exercise can be also found (under the given number in brackets) in the book *Vector Calculus* by J. E. Marsden and A. Tromba.