Exercises
Vector Calculus (MA1103)

Exercise 2

Exercise 1 (2.1: 6)

Let $f(x, y) = 9x^2 + y^2$. Sketch the following.

a) The level curves for f of values $c = 0, 1, 9$

b) The sections of the graph of f in the planes $x = -1$, $x = 0$ and $x = 1$

c) The sections of the graph of f in the planes $y = -1$, $y = 0$ and $y = 1$

d) The graph of f

Exercise 2 (2.1: 9)

Let S be the surface in $\mathbb{R}^3$ defined by the equation $x^2y^6 - 2z = 3$.

a) Find a real-valued function $f(x, y, z)$ of three variables and a constant c such that S is the level set of f of value c

b) Find a real-valued function $g(x, y)$ of two variables such that S is the graph of g

Exercise 3 (2.1: 18)

Draw the level curves (in the xy plane) for $f(x, y) = \frac{x}{y}$ for $c = 0, 1, 2, -1, -2$

Exercise 4 (2.1: 41)

Let $f : \mathbb{R}^2 \setminus \{0\} \to \mathbb{R}$ be given in polar coordinates by $f(r, \theta) = \frac{\cos(2\theta)}{r^2}$. Sketch a few level curves in the xy plane. ($\mathbb{R}^2 \setminus \{0\} := \{x \in \mathbb{R} \mid x \neq 0\}$)

Exercise 5 (2.2: 3a) and 4a))

Compute the limits

i) $\lim_{(x,y)\to(0,1)} x^3y$

ii) $\lim_{(x,y)\to(0,1)} e^xy$

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Exercise 6 (2.2: 17)

Find the limit $\lim_{(x,y)\to(0,0)} \frac{3x^2+3y^2}{\log(x^2+y^2)}$

Hint: Use polar coordinates and L'Hospital

Exercise 7 (2.2: 19)

Show that the subset $B := \{(x, y) \in \mathbb{R}^2 \mid y > 0\}$ of $\mathbb{R}^2$ is open.

Exercise 8 (**A** 2.2: 28) a) Prove that for $x \in \mathbb{R}^n$ and $s < t$: $D_s(x) \subset D_t(x)$.

b) Prove that if U and V are neighborhoods of $x \in \mathbb{R}^n$, then so are $U \cap V$ and $U \cap V$.

c) Prove that the boundary points of an open interval $(a, b) \subset \mathbb{R}$ are the points a and b.

A: This exercise is more theoretical (and might therefore be more difficult).

The exercises can be also found (under the given number in brackets) in the book *Vector Calculus* by J. E. Marsden and A. Tromba.