Exercises
Vector Calculus (MA1103)

Exercise 10

Exercise 1 (6.2: 23)

Let B be the unit ball. Evaluate

$$\iiint_B \frac{1}{\sqrt{2+x^2+y^2+z^2}}\,dx\,dy\,dz.$$

Exercise 2 (6.2: 29)

Integrate $f(x,y,z) = \sqrt{x^2+y^2+z^2}e^{-(x^2+y^2+z^2)}$ over the region W, where W is the solid bounded by the two spheres $x^2+y^2+z^2=a^2$ and $x^2+y^2+z^2=b^2$ with $0<b<a$.

Exercise 3 (6.3: 3)

Find the average of $f(x,y) = y\sin(xy)$ over $D = [0,\pi]\times[0,\pi]$.

Exercise 4 (6.3: 5)

Find the center of mass of the region between $y = x^2$ and $y = x$ if the density is $m(x,y) = x+y$.

Exercise 5 (6.3: 7/8)

A sculptured gold plate D is defined by $0 \le x \le 2\pi$ and $0 \le y \le \pi$ (centimeters) and has mass density $\delta(x,y) = y^2\sin^2(4x)+2$ (grams per square centimeter).

a) If gold sells for \$7 per gram, how much is the gold in the plate worth?

b) What is the average mass density in grams per square centimeter?

Exercise 6 (7.1: 11)

Evaluate the following path integrals $\int_c f(x,y,z)\,ds$, where

a) $f(x,y,z) = e^{\sqrt{z}}$, and $c : [0,1] \to \mathbb{R}^3, t \mapsto (1,2,t^2)$.

b) $f(x,y,z) = yz$, and $c : [1,3] \to \mathbb{R}^3, t \mapsto (t,3t,2t)$.

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Remark

If f is a continuous scalar function on the path a path $c : [a,b] \to \mathbb{R}^n$, then the *average of f along the path c* is given by

$$\frac{1}{l(c)} \int_c f\, ds,$$

where $l(c) := \int_a^b \|c'(t)\|\, dt$ is the length of the path c.

Exercise 7 (7.1: 16b))

Show that the average value of $f(x,y,z) = x^2+y^2+z^2$ along the path $c : [0, 2\pi] \to \mathbb{R}^3, t \mapsto (\cos(t), \sin(t), t)$ is $1 + \frac{4\pi^2}{3}$.

Exercise 8 (7.1: 17)

Find the average y coordinate of the points on the semicircle parametrized by $c : [0, \pi] \to \mathbb{R}^3, \theta \mapsto (0, a\sin(\theta), a\cos(\theta))$, where $a > 0$.

Exercise 9 (7.1: 20)

Show that the path integral of a continuous function $f(x,y)$ over a path C given by the graph of a continuously differentiable function $g(x), a \leq x \leq b$ is given by

$$\int_C f\, ds = \int_a^b f(x, g(x))\sqrt{1 + [g'(x)]^2}\, dx.$$

Conclude that if $g : [a,b] \to \mathbb{R}$ is continuously differentiable, then the length of the graph of g on $[a,b]$ is given by

$$\int_C ds = \int_a^b \sqrt{1 + [g'(x)]^2}\, dx.$$

Exercise 10

Find the length of the graph of $g : [0,1] \to \mathbb{R}, x \mapsto \frac{1}{\sqrt{2}}x^2 + \frac{1}{3}x^3$.

The exercises can be also found (under the given number in brackets) in the book *Vector Calculus* by J. E. Marsden and A. Tromba.