NTNU – Trondheim
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Department of Mathematical Sciences

Examination paper for **MA1103 Vector Calculus**

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Problem 1 Let

$$f(x,y) = \begin{cases} \frac{x^2y^4}{x^4+6y^8} & \text{når } (x,y) \neq (0,0) \\ 0 & \text{når}(x,y) = (0,0). \end{cases}$$

a) Show that $f_x(0,0)$ and $f_y(0,0)$ exist.

b) Show that f is not differentiable at $(0,0)$ by showing that f is not continuous at $(0,0)$.

Problem 2 Find the arclength of the curve with parametrization

$$\mathbf{r}(t) = (\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j} + t^2\mathbf{k}, \qquad \text{for } 0 \leqslant t \leqslant 2\pi.$$

Problem 3 Find the equation for the tangent plane of the surface $x^2 + y^2 - e^{xz} - \sin y = 0$ in the point $(1,0,0)$, and use it to find an approximate value for x in the point on the surface which is near $(1,0,0)$ with $y = z = 1/10$.

Problem 4 The function f is given by $f(x,y) = 2x^2 - x^4 + y^2$.

a) Find all critical points of f, and determine whether these are local maxima, minima, or saddle points.

b) Find the largest and smallest values for f on the curve $x^4 + y^2 = 4$.

Problem 5 Compute the double integral

$$\iint_D (2x + y^2)\, dA$$

where D is the set of all points in the first quadrant lying within the circular disk $x^2 + y^2 \leqslant 4$, but outside the square $0 \leqslant x \leqslant 1$, $0 \leqslant y \leqslant 1$.

Hint: You may find a use for the identity $\sin^2 u = \frac{1}{2}(1 - \cos 2u)$.

Problem 6 Let $\mathbf{F}(x,y,z) = 3x^2y\mathbf{i} + (x^3 + y^3)\mathbf{j}$. Show that $\text{curl}\mathbf{F} = \mathbf{0}$ and find a function f such that $\mathbf{F} = \text{grad} f$.

Problem 7 Let S be the part of the surface $z = 1 + x^2 + y^2$ which lies inside the cylinder $x^2 + y^2 = 1$, and let T be the region between S and the plane $z = 2$.

a) Assume T has a constant mass density $\delta = 1$. Find the mass of T and the coordinates of the mass center.

b) Consider the vector field $\mathbf{F} = f(x, y)\mathbf{i} + xg(x, y)\mathbf{j} + z\mathbf{k}$, where f and g have continuous partial derivatives and satisfy

$$f_x + xg_y = 0$$

for all x, y.

What is the value of the surface integral $\int_S \mathbf{F} \cdot d\mathbf{S}$, where the unit normal vector of S has a negative $\mathbf{k}$ component.