MA1103 FLERDIMENSJONAL ANALYSE, VÅR 2013
ØVING 3

2.1

41. Let $f : \mathbb{R}^2 - \{0\} \to \mathbb{R}$ be given in polar coordinates by $f(r, \theta) = \frac{\cos(2\theta)}{r^2}$. Sketch a few level curves in the xy plane. Here, $\mathbb{R}^2 - \{0\} = \{x \in \mathbb{R}^2 | x \neq 0\}$.

2.2

25.

(1) Can $\frac{\sin(x+y)}{x+y}$ be made continuous by suitably defining it at $(0, 0)$?
(2) Can $\frac{xy}{x^2+y^2}$ be made continuous by suitably defining it at $(0, 0)$?
(3) Prove that $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto ye^x + \sin x + (xy)^4$ is continuous.

2.3

6. Let $f(x, y) = e^{x+y}$. Find the equation for the tangent plane to the graph of f at the point $(0, 0)$.

10. Compute the matrix of partial derivatives

(1) $f(x, y) = (e^x, \sin(xy))$
(2) $f(x, y, z) = (x - y, y + z)$
(3) $f(x, y) = (x + y, x - y, xy)$
(4) $f(x, y, z) = (x + z, y - 5z, x - y)$

11. Find the equation of the tangent plane to $f(x, y) = x^2 - 2xy + 2y^2$ having slope 2 in the positive x direction and slope 4 in the positive y direction.

14. Why should the graphs of $f(x, y) = x^2 + y^2$ and $g(x, y) = -x^2 - y^2 + xy^3$ be called tangent at $(0, 0)$?

16. Use the linear approximation to approximate a suitable function $f(x, y)$ and thereby estimate the following:

(1) $(0.99e^{0.02})^8$
(2) $(0.99)^3 + (2.01)^3 - 6(0.99)(2.01)$
(3) $\sqrt{(4.01)^2 + (3.98)^2 + (2.02)^2}$

26. Evalute the gradient of $f(x, y, z) = \log(x^2 + y^2 + z^2)$ at $(1, 0, 1)$.

2.4

Sketch the curves that are images of the paths in Exercises 1 and 3

1. $c(t) = (\sin t, 4 \cos t)$ for $0 \leq t \leq 2\pi$

3. $c(t) = (2t - 1, t + 2, t)$ for $t \in \mathbb{R}$.

5. Consider the circle C of radius 2, centered at the origin. Find a parametrization for C inducing a counterclockwise orientation and starting at $(2, 0)$.

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