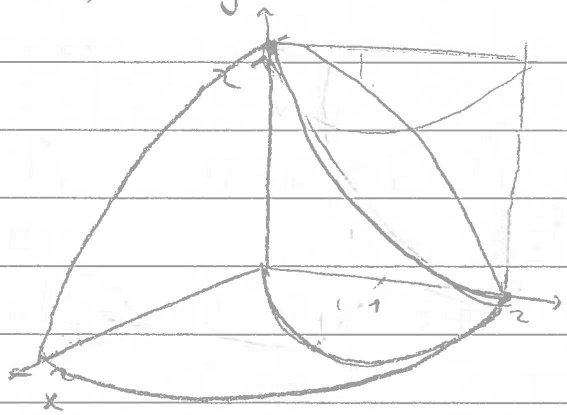
eks: $\mathbb{F}(x,y,z) = [xz,\ yz - z,\ z^2 + y - 4]$

$$S: \begin{cases} x^2 + (y-1)^2 = 1 \\ x^2 + y^2 + z^2 = 4 \\ \text{1-oktant} \end{cases}$$

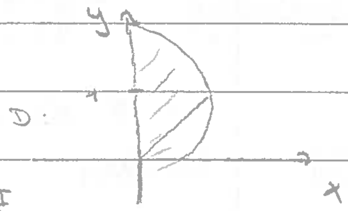


Verifiser divergensteoremet: $\iiint_R \operatorname{div} \mathbb{F}\, dV = \iint_S \mathbb{F} \cdot \hat{\mathbb{N}}\, dS$

① $\iiint_R \operatorname{div} \mathbb{F}\, dV$: $\operatorname{div} \mathbb{F} = z + z + 2z = \underline{4z}$

Grenser: $0 \le z \le \sqrt{4 - x^2 - y^2}$

$\underline{(x,y) \in D}$



$x = r\cos\theta$ $\quad 0 \le \theta \le \frac{\pi}{2}$

$y = r\sin\theta$

$x^2 + (y-1)^2 = 1$

$x^2 + y^2 - 2y + 1 = 1$

$r^2 - 2r\sin\theta = 0$

$r(r - 2\sin\theta) = 0$

$\underline{r = 0} \quad \vee \quad \underline{r = 2\sin\theta}$

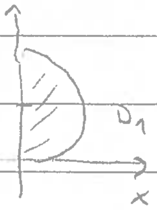
$$\Rightarrow \iiint_R \operatorname{div} \mathbb{F}\, dV = \iint_D \left(\int_{z=0}^{\sqrt{4-x^2-y^2}} 4z\, dz \right) dA$$

$$= \iint_D [2z^2]_0^{\sqrt{4-x^2-y^2}}\, dA = \iint_D 2(4 - x^2 - y^2)\, dA$$

$$= \int_0^{\pi/2} \int_0^{2\sin\theta} 2(4 - r^2)\, r\, dr\, d\theta = 2\int_0^{\pi/2} [2r^2 - \tfrac{1}{4}r^4]_0^{2\sin\theta}\, d\theta$$

$$= \ldots = \underline{\frac{5\pi}{2}}$$

② Fluks: $\iint_S \mathbb{F}\cdot\hat{N}\,dS = \iint_{topp} \mathbb{F}\cdot\hat{N}\,dS + \iint_{bunn} \mathbb{F}\cdot\hat{N}\,dS + \iint_{bakvegg} \mathbb{F}\cdot\hat{N}\,dS + \iint_{front} \mathbb{F}\cdot\hat{N}$

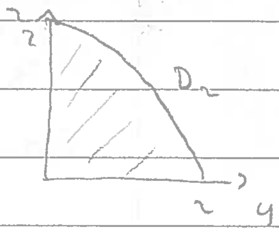


bunn: $\hat{N} = [0,0,-1]$, $\hat{N}\,dS = [0,0,-1]\,dx\,dy$

$\begin{cases} x = x \\ y = y \\ z = 0 \end{cases}$

$$\iint_{bunn} \mathbb{F}\cdot\hat{N}\,dS = \iint_{D_1} 4 - y \underbrace{-z^2}_{=0}\,dx\,dy = \int_0^{\pi/2}\int_0^{2\sin\theta} (4 - r\sin\theta)\,r\,dr\,d\theta$$

$$= \dots = \frac{3\pi}{2}$$



bakvegg: $\begin{cases} x = 0 \\ y = y \\ z = z \end{cases}$ $\hat{N}\,dS = [-1,0,0]\,dy\,dz$

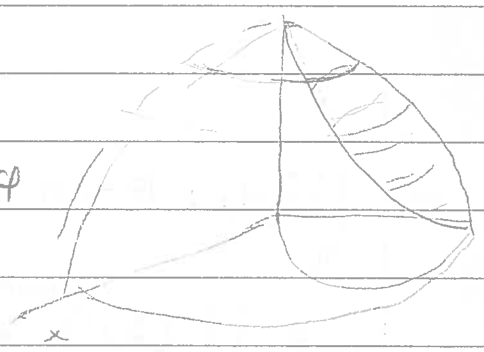
$$\iint_{bakvegg} \mathbb{F}\cdot\hat{N}\,dS = \iint_{D_2} -\underbrace{x}_{=0}z\,dy\,dz = 0$$

topp: $x^2+y^2+z^2=4$ $\begin{cases} x = 2\cos\theta\sin\varphi \\ y = 2\sin\theta\sin\varphi \\ z = 2\cos\varphi \end{cases}$

$\hat{N}\,dS =$

$\hat{N} = \oplus\frac{1}{2}[x,y,z]$

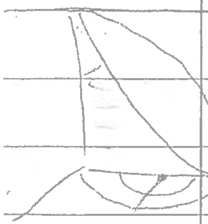
$dS = 2^2\sin\varphi\,d\theta\,d\varphi$



$$\iint_{topp} \mathbb{F}\cdot\hat{N}\,dS$$

$$= \frac{1}{2}\iint_{topp} (x^2z + y^2z \cancel{-zy} + z^3 \cancel{+zy} - 4z)\,dS$$

$$= \frac{1}{2}\iint_{topp} \underbrace{(x^2+y^2+z^2)}_{=4 \text{ på flaten}}z - 4z\,dS = \frac{1}{2}\iint_{topp} 4z - 4z\,dS = 0.$$



$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$0 \le z$: $x^2+y^2+z^2=4 \wedge x^2+y^2=2y$

$z^2 = 4-x^2-y^2 = 4-2y$

$z = \sqrt{4-2y} = \sqrt{4-2(\sin\theta+1)} = \sqrt{2-2\sin\theta}$

(φ : $[0, \frac{\pi}{2}]$

$x^2+y^2+z^2=4$ $\quad x^2+(y-1)^2=1$

$2y+z^2=4$ $\quad x^2+y^2=2y$

$2\cdot 2\sin\theta\sin\varphi + 4\cos^2\varphi = 4$

$\sin\theta\sin\varphi = 1-\cos^2\varphi = \sin^2\varphi$

$\sin\theta = \sin\varphi$ $\quad \varphi = \varphi$)

front: $x^2+(y-1)^2=1$ $\begin{cases} x = 1\cdot\cos\theta \\ y = 1\cdot\sin\theta + 1 \\ z = z \end{cases}$ $\hat{N} = \pm[x, y-1, 0]$ $dS = 1\,d\theta\,dz$

$$\iint_{front} \mathbb{F}\cdot\hat{N}\,dS = \iint_{front} (\underbrace{x^2z + y^2z}_{2yz} - yz - yz + z)\,dS = \iint_{front} z\,dS = \int_{-\pi/2}^{\pi/2}\int_0^{\sqrt{2-2\sin\theta}} z\,dz\,d\theta$$

$$= \int^{\pi/2} [\tfrac{1}{2}z^2]^{\sqrt{\ }}\,d\theta = \dots = \pi$$

$\frac{3\pi}{2} + \pi = \frac{5\pi}{2}$