August 2018

① $B_1 = (0,0)$ $B_2 = (4,0)$

Sentrum $\bar{x} = \dfrac{B_1 + B_2}{2} = (2,0)$

$|PB_1| + |PB_2| = 2a = 8 \Rightarrow a = 4$ (stor halvakse)

Siden $\mathcal{E}$ er en ellipse

$\varepsilon a = |B_1 \bar{x}| \Rightarrow \varepsilon \cdot 4 = 2 \Rightarrow \varepsilon = 1/2$ (eksentrisitet)

$b = a\sqrt{1-\varepsilon^2} = 4\sqrt{1-1/4} = 4\sqrt{3/4} = 2\sqrt{3}$ (lille halvakse)

$$\frac{(x-2)^2}{16} + \frac{y^2}{12} = 1$$

② $\vec{r}(t) = (\sin(t^2-t), \cos(t^2-t))$ $t \geq 0$

$\vec{v}(t) = \vec{r}\,'(t) = ((2t-1)\cos(t^2-t), -(2t-1)\sin(t^2-t))$

$v(t) = \|\vec{v}(t)\| = \sqrt{(2t-1)^2\cos^2(t^2-t) + (2t-1)^2\sin^2(t^2-t)}$

$= |2t-1|$ $t \geq 0$

$\vec{a}(t) = (2\cos(t^2-t) - (2t-1)^2\sin(t^2-t), -2\sin(t^2-t) - \cancel{2}(2t-1)^2\sin^2(t^2-t)$

$a(t) = v'(t) = 2$

3) $f(x) = x^6 - x^5 - 2$

$$\left| \int_0^1 f(x)dx - S_{2n} \right| = \frac{(1-0)^5}{2880 n^4} \left| f^{(4)}(c) \right| < 10^{-2}$$

$f'(x) = 6x^5 - 5x^4$ $\quad f''(x) = 30x^4 - 20x^3$ $\quad f'''(x) = 120x^3 - 60x$

$f^{(4)}(x) = 360x^2 - 120x$

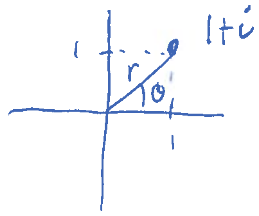
$$\max_{0 \le x \le 1} \left| 360x^2 - 120x \right| = 240$$

$$\frac{240}{2880 n^4} < 10^{-2} \Rightarrow \frac{24000}{2880} < n^4 \Rightarrow n > \sqrt[4]{\frac{24000}{2880}} \approx 1{,}699$$

$\Rightarrow \boxed{n > 2}$

4) $z = 1 + i$

$z = \sqrt{2}\, e^{\pi/4\, i}$



$r = \sqrt{2}$

$\theta = \pi/4$

$$(1+i)^{100} = \left(\sqrt{2}\, e^{\pi/4\, i}\right)^{100} = 2^{50} e^{100\, \pi/4\, i} = 2^{50} e^{25\pi i} =$$

$$= 2^{50} e^{\pi i} = 2^{50}(-1) = -2^{50}.$$

(5) $x_0 = 0 \quad y_0 = 1 \qquad f(x,y) = x y^2$

$x_0' = 0.05 \qquad y_0' = 1 + 0.05\, f(0,1) = 1$

$x_1 = 0.1 \qquad y_1 = 1 + 0.1\, f(0.05, 1) = 1.005$

$x_1' = 0.15 \qquad y_1' = 1.005 + 0.05\, f(0.1, 1.005) \approx 1.0101$

$x_2 = 0.2 \qquad y_2 = 1.005 + 0.1\, f(0.15, 1.0101) \approx 1.0203$

(6) a) $y'' - 4y' - 5y = 0$ (hom. diff lig)

$\lambda^2 - 4\lambda - 5 = 0 \Rightarrow \lambda = 5, -1$

$\boxed{y_h = A e^{5t} + B e^{-t}} \qquad A, B \in \mathbb{R}$

Nå, $y_p = C \cos(2t) + D \sin(2t)$ for $C, D \in \mathbb{R}$

Må finne C og D

$y_p'' - 4y_p' - 5y_p = \cos(2t)$

$\left(-4C \cos(2t) - 4D \sin(2t)\right) - 4\left(-2C \sin(2t) + 2D \cos 2t\right)$

$- 5\left(C \cos(2t) + D \sin(2t)\right) =$

$= (-4C - 8D - 5C) \cos(2t) + (-4D + 8C - 5D) \sin(2t)$

$= (-9C - 8D) \cos(2t) + (-9D + 8C) \sin(2t)$

$$\begin{cases} -9C - 8D = 1 \\ -9D + 8C = 0 \end{cases} \Rightarrow \quad C = -\frac{9}{145}, \quad D = -\frac{8}{145}$$

$$y_p = \frac{-9}{145}\cos(2t) - \frac{8}{145}\sin(2t)$$

så

$$\boxed{y = y_p + y_h = \frac{-9}{145}\cos(2t) - \frac{8}{145}\sin(2t) + Ae^{5t} + Be^{-t}}$$ for $A, B \in \mathbb{R}$

b) $y = \sum_{n=0}^{\infty} a_n x^n \rightsquigarrow y' = \sum_{n=0}^{\infty} a_n \cdot n x^{n-1} \rightsquigarrow y'' = \sum_{n=2}^{\infty} a_n \cdot n(n-1)x^{n-2}$

$y(0) = a_0 = 1 \rightarrow y'(0) = a_1 = 0$

$$\sum_{n=2}^{\infty} a_n\, n(n-1)x^{n-2} + 2x\sum_{n=1}^{\infty} a_n n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n + \sum_{n=1}^{\infty} 2a_n n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$$

$\underline{\underline{n=0}}$ $\quad a_2 \cdot 2 + 2\cancel{a_0}^{\,1} = 0 \rightsquigarrow a_2 = -1$

$\underline{\underline{n=1}}$ $\quad a_3 \cdot 3 \cdot 2 + 2\cancel{a_1}^{\,0} + 2\cancel{a_1}^{\,0} = 0 \rightsquigarrow a_3 = 0$

$\underline{\underline{n \geq 1}}$ $\quad a_{n+2}(n+2)(n+1) + 2a_n \cdot n + 2a_n = 0$

$$a_{n+2}(n+2)(n+1) + 2(n+1)a_n = 0$$

$$\boxed{a_{n+2} = -\frac{2a_n}{n+2}} \qquad \boxed{n \geq 1}$$

$\boxed{a_n = 0}$ $\quad n$ oddetall

$$\boxed{a_{2n} = \frac{(-1)^n 2^{n-1}}{(2n)(2n-2)\cdots 4}} \qquad \boxed{n \geq 1}$$

(7) a) $\sum_{n=0}^{\infty} \underbrace{\frac{1}{n!\,4^n}(x-1)^n}_{a_n}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{1/(n+1)!\,4^{n+1}}{1/n!\,4^n} \cdot \frac{|x-1|^{n+1}}{|x-1|^n} =$$

$$= \lim_{n\to\infty} \frac{1}{(n+1)\cdot 4} \cdot |x-1| = 0$$

Så $\boxed{r=\infty}$ Konvergensområde $\mathbb{R}$

$$\sum_{n=0}^{\infty} \underbrace{(-1)^n \frac{n}{3n+2} x^n}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{n+1}{3(n+1)+2}}{\frac{n}{3n+2}}\right| \cdot |x| = |x| < 1$$

$\boxed{r=1}$

$\underline{x=1}$ $\sum_{n=0}^{\infty} (-1)^n \frac{n}{3n+2} = \infty$

$\underline{x=-1}$ $\sum_{n=0}^{\infty} \frac{n}{3n+2} = \infty$

Konvergensområde $(-1,1)$

(8) $f_n(x) = x^{2n+1} \quad x \in [-1,1]$

$-1<x<1$ $\quad \lim_{n\to\infty} x^{2n+1} = 0$

$x=1$ $\quad \lim_{n\to\infty} f_n(1) = \lim_{n\to\infty} 1^{2n+1} = 1$

$x=-1$ $\quad \lim_{n\to\infty} f_n(-1) = \lim_{n\to\infty} (-1)^{2n+1} = -1$

$$f(x) = \begin{cases} 0 & -1<x<1 \\ 1 & x=1 \\ -1 & x=-1 \end{cases}$$

$f_n(x) \to f(x)$ punktvis, men ikke uniformt fordi $f(x)$ er ikke kontinuerlig.