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① $x^2 + 2x + y^2 = 0$

$$\Downarrow$$

$$\underbrace{x^2 + 2x + 1}_{(x+1)^2} - 1 + y^2 = 0$$

$$\Downarrow$$

$$\boxed{(x+1)^2 + y^2 = 1}$$

sirkel

sentrum $\bar{x} = -1$

Eksentrisitet $\varepsilon = 0$

② a) $y'' + 2y' = 0 \rightsquigarrow \lambda^2 + 2\lambda = 0 \rightsquigarrow \lambda = 0, -2$

$$\rightsquigarrow \boxed{y = Ae^{0x} + Be^{-2x} = A + Be^{-2x}}$$

b) $y_p = (C + Dx)e^{-x} \rightsquigarrow y_p' = ((-C+D) - Dx)e^{-x}$

$$y_p'' = (C - 2D + Dx)e^{-x}$$

$$y_p'' + 2y_p' = (C - 2D + Dx)e^{-x} + 2((-C+D) - Dx)e^{-x}$$

$$= (-C - Dx)e^{-x} = xe^{-x} \rightsquigarrow \begin{array}{l} C = 0 \\ D = -1 \end{array}$$

$$\rightsquigarrow \boxed{y_p = -xe^{-x}}$$

$$\boxed{y = -xe^{-x} + A + Be^{-2x}}$$ generell løsning.

$$y(0) = 0 + A + Be^{0} = \boxed{A + B = 6}$$

$$y' = -e^{-x} + xe^{-x} + -2Be^{-2x}$$

$$y'(0) = -1 + 0 - 2B = -1 \Rightarrow \boxed{B = 0}$$

$$\Rightarrow \boxed{A = 6}$$

$$\boxed{y = -xe^{-x} + 6}$$

c) $$y = \sum_{n=0}^{\infty} a_n x^n \rightsquigarrow y' = \sum_{n=1}^{\infty} n \cdot a_n x^{n-1}$$

$$\rightsquigarrow y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - x\sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=1}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=1}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\left(2a_2 - a_0\right) + \sum_{n=1}^{\infty}\left((n+2)(n+1)\,a_{n+2} - (n+1)a_n\right)x^n = 0$$

$$\boxed{2a_2 - a_0 = 0}$$

$$\boxed{(n+2)(n+1)\,a_{n+2} - (n+1)a_n = 0} \qquad n \geq 1$$

Siden $y(0) = 1 \Rightarrow \boxed{a_0 = 1}$

$y'(0) = 0 \Rightarrow \boxed{a_1 = 0}$

$$a_2 = \frac{a_0}{2} \Rightarrow \boxed{a_2 = 1/2}$$

$$\boxed{a_{n+2} = \frac{(n+1)}{(n+2)(n+1)}\,a_n = \frac{a_n}{n+2}}$$

$$\boxed{a_3 = \frac{0}{3} = 0} \Rightarrow \boxed{a_{2n+1} = 0}$$

$$\boxed{a_4 = \frac{1/2}{4} = 1/8} \Rightarrow \boxed{a_{2n} = \frac{1}{(2n)(2n-2)\cdots 2}}$$

$$n \geq 1$$

(3) $f(x) = 2x + x\sin(x+3) - 5$

$$f'(x) = 2 + \sin(x+3) + x\cos(x+3)$$

$$x_{n+1} = x_n - \frac{2x_n + x_n\sin(x_n+3) - 5}{2 + \sin(x_n+3) + x_n\cos(x_n+3)}$$

$$x_0 = 3$$

$$\boxed{x_1 = 3 - \frac{6 + 3\sin 6 - 5}{2 + \sin 6 - 3\cos 6} \approx 3.1395}$$

$$\boxed{x_2 \approx 3.803}$$

(4) $\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 + R_5(x)$

$$\sin x = x - \frac{1}{3!}x^3 + R_4(x)$$

$$\ln(1+x) = x + R_2(x)$$

$$\lim_{x \to 0} \frac{\left(1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 + R_5(x)\right) - 1 + \frac{1}{2}x\left(x - \frac{1}{3!}x^3 + R_4(x)\right)}{(x + R_2(x))^4} =$$

$$= \lim_{x \to 0} \frac{\cancel{-\frac{1}{2}x^2} + \frac{1}{4!}x^4 + R_5(x) + \cancel{\frac{1}{2}x^2} - \frac{1}{2\cdot 3!}x^4 + \frac{1}{2}x R_4(x)}{x^4 + P_5(x)}$$

polynomet av grad større enn 4.

$$= \lim_{x \to 0} \frac{-\frac{1}{12}x^4 + q_5(x)}{x^4 + P_5(x)} = \boxed{\frac{-1}{12}}$$

b) $\displaystyle\sum_{n=0}^{\infty} \underbrace{\frac{1}{n^2 4^n} x^{2n}}_{r_n}$

$$\lim_{n \to \infty} \frac{r_{n+1}}{r_n} = \lim_{n \to \infty} \frac{\frac{x^{2n+2}}{(n+1)^2 4^{n+1}}}{\frac{x^{2n}}{n^2 4^n}} = \lim_{n \to \infty} \frac{n^2 x^2}{(n+1)^2 4} =$$

$$= \frac{x^2}{4} \rightsquigarrow \frac{x^2}{4} < 1 \rightsquigarrow x^2 < 4 \rightsquigarrow \boxed{-2 < x < 2}$$

$\boxed{x = 2} \rightsquigarrow \displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2 4^n} 2^{2n} = \sum_{n=0}^{\infty} \frac{1}{n^2} < \infty$

$\boxed{x = -2} \rightsquigarrow \displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2 4^n} (-2)^{2n} = \sum_{n=0}^{\infty} \frac{1}{n^2} < \infty$

⑤ $y' = f(x,y) = \frac{-x}{y^2}$ $\quad y(0) = 1$

$x_0 = 0, \; y_0 = 1, \quad h = 0.1$

$x_0' = 0.05,$

$y_0' = 1 + 0.05 \cdot f(0,1) = 1 + 0.05 \cdot 0 = 0 + 1 = 1$

$\boxed{y_1 = 1 + 0.1 \cdot f(0.05, 1) = 1 + 0.1 \cdot (-0.05) = 0.995}$

$x_1' = 0.1 + 0.05 = 0.15$

$y_1' = 0.995 + 0.05 f(0.1, 0.995) = 0.9899$

$\boxed{y_2 = 0.995 + 0.1 \, f(0.1, 0.9899) = 0.9848}$

⑥ $\lim_{n\to\infty} \overbrace{\frac{1}{n + n(x-n)^2}}^{f_n} = 0 \quad \forall x \in \mathbb{R}$

$f_n \longrightarrow 0$ punktvis

$d(f_n, 0) = \frac{1}{n} \longrightarrow 0 \quad n \to \infty$

dvs $f_n \longrightarrow 0$ uniformt.