Norges teknisk–naturvitenskapelige universitet
Department of Mathematical Sciences

MA1102
Grunnkurs i analyse II
Vår 2023

Øving 8

1 For each of the following function sequences $(f_n)_n$ and associated domains, determine if the sequence is pointwise convergent to some function f on that domain and if so, is it also uniformly convergent to f?

a) $I = \mathbb{R}, \quad f_n(x) = xe^{-nx^2}.$

b) $I = [-1, 1] \quad f_n(x) = x^{2n+1}.$

c) $I = \mathbb{R}, \quad f_n(x) = \frac{1}{n+n(x-n)^2}.$

d) $I = (0, \infty), \quad f_n(x) = \frac{nx}{1+n^2x^2}.$

e) $I = [-1, 1], \quad f_n(x) = \frac{n^2}{1+n^2x^2+n^2}.$

f) $I = [0, \infty), \quad f_n(x) = ne^{-nx}.$

g) $I = \mathbb{R}, \quad f_n(x) = \frac{nx}{n+x}.$

Hint: If $(f_n)_n$ is a sequence of continuous functions which converge uniformly to f, then f must also be continuous.

2 Prove that if the function sequence $(f_n)_n$ is uniformly convergent to f on $I \subset \mathbb{R}$, then the sequence also converges pointwise. Also give an example showing that the converse is not true.

3 Prove that $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

4 Find a continuous function $f : (0, \infty) \to \mathbb{R}$ which is not uniformly continuous.