Norges teknisk–naturvitenskapelige universitet
Department of Mathematical Sciences

MA1102
Grunnkurs i analyse II
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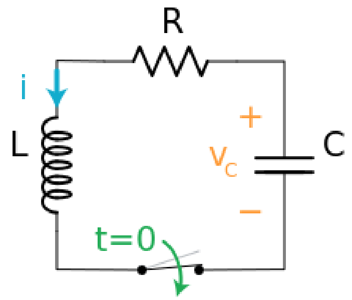
Øving 13

1 Find values of the constants $a_0, a_1, a_2, \dots$ such that $y_p(x) = \sum_{n=0}^{\infty} a_n x^n$ is a particular solution to the differential equation

$$y' - y = 0, \qquad y(0) = 1.$$

2 In electrical engineering, a fundamental object is the resistor-inductor-capacitor (RLC) circuit, see the figure below.



Figur 1: An RLC circuit

We will be interested in determining the current as a function of time, $i(t)$. Without going into the electrical details, we state that the voltage differences for each of the three elements are given by

$$v_L = L\frac{di}{dt},$$
$$v_R = -iR,$$
$$v_C = \frac{1}{C}\int_0^t -i(s)ds.$$

By peering deep into our inner electrical engineer and the figure, we see that it must be the case that

$$v_L - v_R - v_C = 0.$$

a) Translate the above equality into a linear second order homogeneous differential equation and solve it. Since we do not know the specific values of R, L and C a full simplification may not be possible.

b) At time $t = 0$ there is no current in the system and the voltage over the capacitor is some undetermined amount $v_C(0) = V_0$. Translate this into initial conditions

for the ODE we just solved and use it to fix the two constants.
Hint: Put $t = 0$ *into the differential equation.*

3 In quantum mechanics, the *wavefunction* ψ is of great interest as is describes the state of a quantum system. To find the wavefunction of a system, we solve the *Schrödinger equation*

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x,t)\Psi = i\hbar\frac{\partial \Psi}{\partial t}.$$

One of the simpler incarnations of this equation is that for a particle with mass m stuck in a one-dimensional box of width L. In this case, the Schrödinger equation can be simplified to

$$-\frac{\hbar^2}{2m}\frac{d^2 \psi}{dx^2} = E\psi, \qquad \psi(0) = \psi(L) = 0$$

where E is the energy of the particle.

a) Rewrite the simplified Schrödinger equation into a linear second order homogeneous differential equation and find the general solution without taking into account the boundary conditions.

b) Apply the boundary conditions to simplify the solution. You will find that a non-zero solution only exists for certain values of the energy E, what are these?
Hint: $\sin(\pi n) = 0$ *for all integer* n.

c) Wavefunctions are required to satisfy the normalization condition

$$\int_0^L |\psi(x)|^2\, dx = 1.$$

Use this to get rid of the final undetermined coefficient in your solution.

Fun fact: A slightly more complex but still similar version of this takes place in very small 3-dimensional boxes inside your (new and expensive) TV called quantum dots. By changing the size of the small boxes (corresponds to changing the parameter L*) the energy* E *changes which determines which color the quantum dots output.*