Norges teknisk–naturvitenskapelige universitet
Department of Mathematical Sciences

MA1102
Grunnkurs i analyse II
Vår 2023

Øving 10

1 **a)** Compute the antiderivative

$$\int \frac{1}{1+x^2}\, dx.$$

Hint: Substitute $x = \tan(\theta)$

b) Write $\frac{1}{1+x^2}$ as a power series. Where does it converge?

c) Use Abel's theorem to compute the sum

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}.$$

2 Recall that the Taylor series of a function f around a point a is given by

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

a) Compute the Taylor coefficients of $\sin(x)$ and $\cos(x)$ around $x = 0$ and write up the Taylor series. For which x do the Taylor series converge?

b) Prove that the Taylor series converge uniformly to the trigonometric functions on compact subsets of $\mathbb{R}$. Why does this imply pointwise equality on all of $\mathbb{R}$?

3 For $f(x) = \arctan(x)$, compute

$$f^{(11)}(0).$$

Hint: Exercise 1.

4 Compute the power series for

$$f(x) = \int_0^x \ln\left((28-t^3)^2\right) dt$$

and compute the radius of convergence.