

Numeriske metoder for differensialligninger

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Eulers metode

$$y' = f(x, y), \quad y(x_0) = y_0$$

Velg h liten:

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

så får vi

$$x_n = x_0 + nh$$

$$y_n = y(x_n) + O(nh^2) = y(x_n) + O((x_n - x_0)h)$$

Lokal feil blir $O(h^2)$, total feil blir $O(h)$.

Forbedret Euler

$$y' = f(x, y), \quad y(x_0) = y_0$$

Velg h liten:

$$x_{n+1} = x_n + h$$

$$u_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{1}{2}h(f(x_n, y_n) + f(x_{n+1}, u_{n+1}))$$

så får vi

$$x_n = x_0 + nh$$

$$y_n = y(x_n) + O(nh^3) = y(x_n) + O((x_n - x_0)h^2)$$

Lokal feil blir $O(h^3)$, total feil blir $O(h^2)$.

Runge–Kutta (RK4)

$$y' = f(x, y), \quad y(x_0) = y_0$$

Velg h liten:

$$x_{n+1} = x_n + h$$

$$p_n = f(x_n)$$

$$q_n = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hp_n\right)$$

$$r_n = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hq_n\right)$$

$$s_n = f(x_n + h, y_n + hr_n)$$

$$y_{n+1} = y_n + \frac{1}{6}h(p_n + 2q_n + 2r_n + s_n)$$

Lokal feil blir $O(h^5)$, total feil blir $O(h^4)$.