NTNU
Norwegian University of
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Department of Mathematical Sciences

Examination paper for **MA1101 Grunnkurs i Analyse I**

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Originalen er:

1-sidig ☐ **2-sidig** ☒

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Date Signature

Problem 1 Let

$$f : \mathbb{R} \to \mathbb{R}, \qquad f(x) = xe^{-x} + 1.$$

a) Find the second degree Taylor polynomial of f at the point $x_0 = \ln 2$.

b) What are the global minimum and the global maximum of f?

Problem 2 Decide if each of the following statements is true or false. (You do not need to justify your answers).

a) Every Riemann–integrable function $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$.

b) Every convergent sequence $(a_n)_{n=1}^{\infty} \subseteq \mathbb{R}$ is also bounded.

c) Every continuous function $f : [0, 1] \to \mathbb{R}$ is also uniformly continuous.

d) $\lim_{x\to 0} \dfrac{\sin x}{\sinh x} = 1.$

e) If $f : \mathbb{R} \to \mathbb{R}$ is twice differentiable on $x_0 \in \mathbb{R}$ and $f''(x_0) = 0$, then x_0 is an inflection point (saddle point) of f.

Problem 3 Evaluate the following integrals.

$$\int_2^3 \frac{1}{x(x-1)^2} \mathrm{d}x, \quad \int_0^{\pi} x \sin x \, \mathrm{d}x, \quad \int_0^1 \frac{1}{\sqrt{x^2+1}} \, \mathrm{d}x.$$

Problem 4 Solve the integral equation

$$y(x) = \int_0^x 2t \cdot (1 + y^2(t)) \, \mathrm{d}t.$$

Problem 5 Evaluate the following series.

a)

$$\sum_{n=2}^{\infty} \frac{1}{(n+1)(n-1)} \, .$$

b)

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}.$$

Problem 6 Does the integral

$$\int_{-\infty}^{+\infty} \frac{x^2}{x^4+3} \mathrm{d}x$$

converge or diverge? Justify your answer.

Problem 7 Find the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n(k+n)}.$$

Problem 8 Let

$$A = [-1, 0) \cup (1, 2).$$

a) Find $\sup A$. Justify your answer.

b) Find (without proof) $\inf A$, $\max A$ and $\min A$.

Problem 9 Suppose the series $\sum_{n=1}^{\infty} a_n$ converges. Prove that

$$\lim_{n\to\infty} (a_{n-1}a_{n+1} - a_n^2) = 0.$$

Justify your answer. *(Anything that was taught in the lectures can be used without proof.)*

Problem 10 Find the values of the real number $\lambda \in \mathbb{R}$ for which the integral

$$\int_0^{\infty} \frac{1}{(e^x - e^{-x})^{\lambda}} \mathrm{d}x$$

converges and diverges, respectively. Justify your answer.