Norwegian University of Science and Technology
Department of Mathematical Sciences

MA1101 Basic Calculus I
Fall 2022

Exercise set 9
Deadline: Oct. 28

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

1 Simplify the expressions below.

a) $e^{2\ln\cos(x)} + \left(\ln e^{\sin(x)}\right)^2$

b) $\log_\pi(1-\cos(x)) + \log_\pi(1+\cos(x)) - 2\log_\pi \sin(x)$

c) $\sinh(\ln x)$, where the hyperbolic sine function is defiend by

$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \quad x \in \mathbb{R}.$$

2 Let P_n denote the partition of the given interval $[a, b]$ into n subintervals of equal length $\Delta x_i = \frac{b-a}{n}$. Evaluate the lower Riemann sum $L(f, P_n)$ and the upper Riemann sum $U(f, P_n)$ for the given functions f and the given values of n.

a) $f(x) = x$ on $[0, 2]$, with $n = 8$

b) $f(x) = e^x$ on $[-2, 2]$, with $n = 4$

3 Express the given limit as a definite integral.

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\sqrt{\frac{i}{n}}$

b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \ln\left(1 + \frac{2i}{n}\right)$

4 Evaluate the integrals below.

a) $\int_{-2}^{2} (x+2)\,\mathrm{d}x$

b) $\int_{1}^{2} \left(\frac{2}{x^3} - \frac{x^3}{2}\right)\mathrm{d}x$

c) $\int_{-4}^{4} (e^x - e^{-x})\,\mathrm{d}x$

5 Find the indicated derivatives below.

a) $\frac{\mathrm{d}}{\mathrm{d}t} \int_t^3 \frac{\sin(s)}{s} \, \mathrm{d}s$

b) $\frac{\mathrm{d}}{\mathrm{d}x} F(\sqrt{x})$, if $F(t) = \int_0^t \cos(s^2) \, \mathrm{d}s$

6 Find the following integral of the piecewise continuous function

$$\int_0^{\frac{3\pi}{2}} |\cos(x)| \, \mathrm{d}x.$$

7 Let $f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } 1 < x \leq 2 \end{cases}$. Show that f is integrable on $[0, 2]$ and find the value of $\int_0^2 f(x) \, \mathrm{d}x$.

8 Use the mean-value theorem for integrals to calculate

$$\lim_{n \to \infty} \int_n^{n+p} \frac{\sin(x)}{x} \, \mathrm{d}x, \quad p, n \text{ are natural numbers.}$$

9 Let $f \in C([0, 1], \mathbb{R})$ be differentiable on the open interval $(0, 1)$, and assume that $4 \int_{\frac{3}{4}}^1 f(x) \, \mathrm{d}x = f(0)$. Prove that there exists $c \in (0, 1)$, such that $f'(c) = 0$.

Hint: Consider the mean-value theorem for integrals first, then use Rolle's theorem.