MA1101 Basic Calculus I
Fall 2022

Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 7
Deadline: Oct. 14

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

1 Show that the functions f below are bijective, and calculate the inverse functions f^{-1}. Specify the domains and ranges of f^{-1}.

a) $f\colon [1,\infty) \to \mathbb{R}, \quad x \mapsto \sqrt{x-1}$

b) $f\colon (-\infty,-1) \cup (-1,\infty) \to \mathbb{R}, \quad x \mapsto \dfrac{x}{1+x}$

2 Find the value of x when

$$2^{x^2-3} = 4^x.$$

3 If functions f and g have respective inverse f^{-1} and g^{-1}, show that the composite function $f \circ g$ has inverse $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

4 Prove that a function $f : \mathbb{R} \to \mathbb{R}$ which is strictly increasing is injective.

5 Find the sum of the given series below, or show that the series diverge.

a) $\displaystyle\sum_{k=0}^{\infty} \frac{2^{k+3}}{e^{k-3}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \cdots$

Hint: Use that $\dfrac{1}{(2n-1)(2n+1)} = \dfrac{1}{2}\Big(\dfrac{1}{2n-1} - \dfrac{1}{2n+1}\Big)$.

c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$

6 Find the required Taylor series representations of the functions below.

a) $f(x) = \frac{1}{4-x^2+2x}$ about $x = 1$. *(Hint: Make the substitution $t = x - 1$)*

b) $f(x) = \cos^2(x)$ about $\dfrac{\pi}{8}$

7 Decide whether the given statements are TRUE or FALSE. If it is TRUE, prove it. If it is FALSE, give a counterexample.

a) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges to infinity.

b) If $a_n \geq c > 0$ for every n, then $\sum_{n=1}^{\infty} a_n$ diverges to infinity.

c) If $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (a_n)^2$ converges.

8 Prove that if $\{x_n\}_n$ and $\{y_n\}_n$ are two Cauchy sequences in $\mathbb{R}$, the sequence $\{x_n+y_n\}_n$ is also a Cauchy sequence in $\mathbb{R}$.