Norwegian University of Science
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Department of Mathematical
Sciences

MA1101 Basic Calculus I
Fall 2022

Exercise set 6
Deadline: Oct. 7

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

1 Classify the critical points of the function

$$f\colon \mathbb{R} \to \mathbb{R}, \quad x \mapsto x(x^2-1)^2.$$

(That is, decide if they are local/global maxim/minima or not.)

2 Sketch the graph of the function

$$f\colon \mathbb{R}\backslash\{\pm 1\} \to \mathbb{R}, \quad x \mapsto \frac{x^3}{x^2-1}.$$

Make a table with the sign of f' and f'', and the corresponding behavior of f. Describe the asymptotes of f (see Chapter 4.6 in Adams for background on asymptotes if necessary).

3 All 80 rooms in a motel will be rented each night if the manager charges 40 NOK or less per room. If he charges $(40+x)$ NOK per room, then $2x$ rooms will remain vacant. If each rented room costs the manager 10 NOK per day and each unrented room 2 NOK per day in overhead, how much should the manager charge per room to maximize his daily profit?

4 Find the linearization of the given function about the given point.

$$f\colon \mathbb{R} \to \mathbb{R}, \quad x \mapsto \sqrt{3+x^2} \quad \text{about} \quad x = 1.$$

5 Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ be a polynomial. Show that the Taylor series around $x_0 = 0$ of $p(x)$ is equal to $p(x)$.

6 Find the fourth order Taylor polynomial of the function $f\colon \mathbb{R} \to \mathbb{R}$, $x \mapsto e^x$ at the point $x_0 = \ln 2$.

7 Calculate

a) $\lim_{x\to\infty} \dfrac{3x + \ln(x) + 2x^3}{x^3}$.

b) $\lim_{x\to 0} \dfrac{\tan(x) - x}{x^2 \sin(x)}$.

8 Calculate

a) $\lim_{x\to 0} \dfrac{\sin(ax)}{\sin(bx)}$, for $a, b > 0$.

b) $\lim_{h\to 0} \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$, given that f is twice derivable.

9 Sketch the graph of the function

$$f : \mathbb{R} \setminus \{0\}, \quad x \mapsto \frac{e^{|x|}}{|x|}$$

and describe any asymptotes.