Norwegian University of Science
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Department of Mathematical
Sciences

MA1101 Basic Calculus I
Fall 2022

Exercise set 5
Deadline: Sept. 30

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

1 Let $y\colon \mathbb{R} \to \mathbb{R}$, $x \mapsto \frac{1}{a+bx}$, where $a \neq 0$, $b \neq 0$.

a) Find $\frac{\mathrm{d}^3 y}{(\mathrm{d}x)^3}(x)$.

b) Find a general formula for $\frac{\mathrm{d}^n y}{(\mathrm{d}x)^n}(x)$ for $n \in \mathbb{N}$. Give an argument for it.

2 Show that

$$\sin(2x) > x \quad \text{when} \quad 0 < x < \frac{\pi}{8}.$$

Prove it analytically, not graphically.

3 The volume V in a water tank can be described using the formula

$$V(t) = 350(20 - t)^2 \,\mathrm{L}, \quad t \geq 0.$$

The relevant physical unit is liters (L), and we count $t = 0$ as the start time. The time t is measured in minutes. How much water flows out per minute after 5 minutes; after 15 minutes?

4 Show that the function $f\colon \mathbb{R} \to \mathbb{R}$, $x \mapsto x^3$ is increasing on the whole real line even though $f'(x)$ is not positive at every point.

5 Use (formal) implicit differentiation to find the tangent to the curve (x, y) when

$$x^2 + y^2 + 2xy + x = 1, \quad (x, y) = (0, 1).$$

6 Let $z\colon \mathbb{R}\backslash A \to \mathbb{R}$, $x \mapsto \tan\left(\frac{x}{2}\right)$, where $A = \{x\colon x = \frac{\pi}{4} + \frac{k\pi}{2},\ k \in \mathbb{Z}\}$. Show that

$$\frac{\mathrm{d}x}{\mathrm{d}z} = \frac{2}{1+z^2}, \quad \sin(x) = \frac{2z}{1+z^2}, \quad \text{and} \quad \cos(x) = \frac{1-z^2}{1+z^2}.$$

7 Determine

$$\int \frac{2x}{\sqrt{x^2+1}}\,\mathrm{d}x.$$

8 Let $f\colon \mathbb{R}\backslash\{0\} \to \mathbb{R}$, $x \mapsto x - \frac{4}{x}$. Show that $f(-1) = f(4)$, but that there is no point $c \in (-1, 4)$ such that $f'(c) = 0$. Why does this not contradict Rolle's theorem?