MA1101 Basic Calculus I
Fall 2022

Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 4
Deadline: Sep. 23

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

1 Find the equations of the form $y = kx + m$ tangent at x_0 to

a) $y = \frac{1}{\sqrt{x}}, \quad x_0 = 9$

b) $y = \frac{1}{x^2+1}, \quad x_0 = 0.$

2 Write down a plausible definition for

$$\lim_{x \to a^-} f(x) = -\infty.$$

3 Show that if $f : [0, \infty) \to \mathbb{R}$ satisfies $f(x) > 0$ for all $x \geq 0$ and $\lim_{x \to \infty} f(x) = 0$, then f attains its maximum.
Hint: We want to be able to restrict our attention to a compact subset of $[0, \infty)$ *and apply the extreme value theorem there.*

4 Calculate, using the definition, the derivative of the following functions

a) $f \colon \mathbb{R} \to \mathbb{R}, \qquad x \mapsto 1 + 4x - 5x^2$

b) $g \colon \mathbb{R} \backslash \{0\} \to \mathbb{R}, \qquad x \mapsto x + \frac{1}{x}$

c) $h \colon (-1, \infty) \to (0, \infty), \qquad x \mapsto \frac{1}{\sqrt{1+x}}.$

Also determine the maximal domain for the functions f', g' and h'.

5 Show that the curve $y = x^2$ and the straight line $x + 4y = 18$ intersect at right angles at one of their two intersection points.

Hint: Find the product of their slopes at their intersection points.

6 Use the factoring of a difference of cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}x^{\frac{1}{3}} = \frac{1}{3}x^{-\frac{2}{3}}, \quad x \neq 0,$$

with the help of the definition of derivative.

7 Suppose f is continuous on [-2, 3] and $f(-2) = 5$, $f(3) = -5$. Show that f has a zero somewhere in $(-2, 3)$.

8 Find

a) $\frac{\mathrm{d}}{\mathrm{d}x}(2 + |x|^3)^{\frac{1}{3}}$

b) $\frac{\mathrm{d}}{\mathrm{d}t}f\Big(2 - 3f(4 - 5t)\Big), \quad f\colon \mathbb{R} \to \mathbb{R}$ arbitrary

c) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sqrt{x^2-1}}{x^2+1}\right)\Big|_{x=-2}$

and state in **a)**, **b)** and **c)** for which values of x applies.

9 Assume $f(x)$ is continuous at $x = 0$. For the following statements, if TRUE, give reasons; if FALSE, give a counterexample.

a) If $\lim_{x \to 0} \frac{f(x)}{x}$ exists, then $f(0) = 0$.

b) If $\lim_{x \to 0} \frac{f(x) + f(-x)}{x}$ exists, then $f(0) = 0$.

c) If $\lim_{x \to 0} \frac{f(x)}{x}$ exists, then $f'(0)$ exists.

d) If $\lim_{x \to 0} \frac{f(x) - f(-x)}{x}$ exists, then $f'(0)$ exists.