



Norwegian University of Science
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Department of Mathematical
Sciences

MA1101 Basic Calculus I
Fall 2022

Exercise set 2

The deadline for this exercise is Friday Sep. 9. You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

- 1 Which of the following sequences are bounded above/below (give a bound) and/or increasing/decreasing (give an argument)?

a) $(x_n)_{n \geq 1} = (n)_{n \geq 1}$

b) $(1, 1, 1, 1, 1, 1, 1, \dots)$

c) $(1, -1, 1, -2, 1, -3, 1, \dots)$

d) The sequence $(x_n)_n$ given by

$$x_1 = \sqrt{2}, \quad x_{n+1} = \sqrt{2 + x_n}, \quad n \geq 1.$$

Hint: The sequence is convergent with $\lim_{n \rightarrow \infty} x_n = 2$.

- 2 Find supremum, infimum and the limit of the sequence $(x_n)_{n \in \mathbb{N}}$ given by $x_n = \frac{(-1)^n}{n}$. Use the definitions to show this rigorously.

- 3 Evaluate the limit or explain why it does not exist.

a) $\lim_{x \rightarrow -1} \frac{(x+1)^3}{x+1}$

b) $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{1}{x^2-4} \right)$

c) $\lim_{x \rightarrow 0} \frac{|3x-1| - |3x+1|}{x}$

- 4 Evaluate the limit or explain why it does not exist.

a) $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x^2}\right)$

b) $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$

5 Use ε/δ to verify the indicated limit.

a) $\lim_{x \rightarrow 1} (2x + 2) = 4$

b) $\lim_{x \rightarrow 0} x^2 = 0$

6 Given a function

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0, \end{cases}$$

state where in its natural domain it is continuous, and where it is discontinuous.

7 Find k so that

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 3 \\ k - x^2 & \text{if } x > 3 \end{cases}$$

is a continuous function.

8 Prove that if there are two limits a and b to the same real sequence $(x_n)_n$, then $a = b$.