

HAND-IN ASSIGNMENT, MA1101

Deadline: Sunday October 31, 23.59.

Solutions should be clear, written on paper, tablet or by other means in English or Norwegian, and handed in as PDFs in ovsys.

Choose *two* topics, and write systematically about both (a) and (b) in each of these two topics. The expected extent is approximately two handwritten pages per topic, but will vary with your way of writing. Express yourself rigorously, explaining to someone who knows mathematics, but not the topic. You may work and hand in individually or in pairs of two, but you should be ready to answer questions individually about the handed-in material. The solution should be signed with your full name, as well as the name of your possible coworker.

The hand-in assignment is weighted as 30% of the complete course grade.

Topic 1. Functions.

- (a) Describe the basic theory of functions, including the concepts of domains, codomains, images, graphs and point values, injectivity, surjectivity and invertibility.
- (b) Use the example of

$$f(x) = \frac{1}{|x|(1+x^2)}$$

to illustrate the theory in (a), showing which concepts apply to it (it is deliberate here, that no domain of definition is given).

Topic 2. Continuity.

- (a) Describe the basic theory of continuity, including the ε/δ -definition, left- and right-continuity, and the extremal and intermediate value theorems. Give a very simple example or illustration of each of the concepts.
- (b) Show using the ε/δ -definition that $f: \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

is continuous at $x = 1$, but discontinuous at $x = 0$

Topic 3. Extremal values.

(a) Describe the basic theory of extremal values, including critical and singular points, the mean-value theorem, local and global extrema, and the second derivative test.

(b) Find any local or global maxima of $f: [0, 2\pi) \rightarrow \mathbb{R}$,

$$f(x) = x(x - \pi) \sin(x).$$

Hint: use your knowledge about the theory and the factors of f .

Topic 4. Differentiation and approximation.

(a) Describe the concepts of differentiation, linearisation of a function around a point, and some higher order approximation (Taylor's theorem) including the remainder term.

(b) Find the linearisation of $f(x) = \cos(x)e^x$ around the point $x = \pi$, and give an estimate for the error between $f(x)$ and the linearisation for $x \in [\frac{\pi}{2}, \frac{3\pi}{2}]$.