



Norwegian University of Science  
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Department of Mathematical  
Sciences

MA1101 Basic Calculus I  
Fall 2021

**Exercise set 5**  
**Deadline: Oct. 1**

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

You can pose questions regarding homework or lecture etc. on the discussion forum Digital Mattelab, see <https://wiki.math.ntnu.no/ma1101/2021h/start>.

1 Let  $y: \mathbb{R} \rightarrow \mathbb{R}$ ,  $x \mapsto \frac{1}{a+bx}$ , where  $a \neq 0$ ,  $b \neq 0$ .

a) Find  $\frac{d^3y}{(dx)^3}(x)$ .

b) Find a general formula for  $\frac{d^n y}{(dx)^n}(x)$  for  $n \in \mathbb{N}$ . Give an argument for it.

2 Show that

$$\sin(2x) > x \quad \text{when} \quad 0 < x < \frac{\pi}{8}.$$

Prove it analytically, not graphically.

3 The volume  $V$  in a water tank can be described using the formula

$$V(t) = 350(20 - t)^2 \text{ L}, \quad t \geq 0.$$

The relevant physical unit is liters (L), and we count  $t = 0$  as the start time. The time  $t$  is measured in minutes. How much water flows out per minute after 5 minutes; after 15 minutes?

4 Show that the function  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $x \mapsto x^3$  is increasing on the whole real line even though  $f'(x)$  is not positive at every point.

5 Use (formal) implicit differentiation to find the tangent to the curve  $(x, y)$  when

$$x^2 + y^2 + 2xy + x = 1, \quad (x, y) = (0, 1).$$

- 6 Let  $z: \mathbb{R} \setminus A \rightarrow \mathbb{R}$ ,  $x \mapsto \tan\left(\frac{x}{2}\right)$ , where  $A = \{x: x = \frac{\pi}{4} + \frac{k\pi}{2}, k \in \mathbb{Z}\}$ . Show that

$$\frac{dx}{dz} = \frac{2}{1+z^2}, \quad \sin(x) = \frac{2z}{1+z^2}, \quad \text{and} \quad \cos(x) = \frac{1-z^2}{1+z^2}.$$

- 7 Determine

$$\int \frac{2x}{\sqrt{x^2+1}} dx.$$

- 8 Let  $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ ,  $x \mapsto x - \frac{4}{x}$ . Show that  $f(-1) = f(4)$ , but that there is no point  $c \in (-1, 4)$  such that  $f'(c) = 0$ . Why does this not contradict Rolle's theorem?

- 9 Locate any inflection points of the given function below.

$$f(x) = \frac{x^3}{3} - 4x^2 + 12x - \frac{25}{3}, \quad x \in \mathbb{R}.$$