Norwegian University of Science and Technology
Department of Mathematical Sciences

MA1101 Basic Calculus I
Fall 2021

Exercise set 13
Deadline: Nov. 28

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

You can pose questions regarding homework or lecture etc. on the discussion forum Digital Mattelab, see https://wiki.math.ntnu.no/ma1101/2021h/start.

1 Compute $\lim_{n\to\infty} x_n$ for the following sequences or explain why the limit does not exist.

a)

$$x_n = \begin{cases} \frac{n+2}{n+1}, & n \text{ is odd} \\ \frac{n}{n+1}, & n \text{ is even} \end{cases}$$

b)

$$x_n = \begin{cases} \frac{n}{1+n}, & n \text{ is odd} \\ \frac{n}{1-n}, & n \text{ is even} \end{cases}$$

c)

$$x_n = \begin{cases} 1 + \frac{1}{n}, & n \text{ is odd} \\ (-1)^n, & n \text{ is even} \end{cases}$$

d)

$$x_n = \begin{cases} 1, & n < 10^6 \\ \frac{1}{n}, & n \geq 10^6 \end{cases}$$

2 Evaluate the integral below.

a)

$$\int \frac{1}{5 - x^2}\, dx$$

b)

$$\int \frac{1}{(a^2 + x^2)^{\frac{3}{2}}}\, dx$$

c)

$$\int_0^{\frac{\pi}{2}} \frac{1}{\cos x}\,dx$$

3 Let

$$f(x) = \begin{cases} \frac{x}{3}\sin(\frac{2}{x}), & x < 0, \\ a, & x = 0, \\ \frac{2}{x}\sin(\frac{x}{3}), & x > 0. \end{cases}$$

Show that f is discontinuous at $x = 0$ no matter what the value of a is.

Hint: Consider $\lim_{x\to 0+} f(x)$ *and* $\lim_{x\to 0-} f(x)$.

4 Find the sum of the given series, or show that the series diverges.

a)

$$\sum_{n=0}^{\infty} \frac{1}{e^n}$$

b)

$$\sum_{n=0}^{\infty} \frac{n}{n+2}$$

5 Locate any inflection points of the given function below.

$$f(x) = \int_0^x (1-t)\arctan(t)\,\mathrm{d}t.$$

6 Use the mean value theorem for integrals to calculate the limit below.

$$\lim_{n\to\infty} \int_0^1 \frac{x^n}{1+x}\,dx.$$

7 Let $f \in C([0,3], \mathbb{R})$ be differentiable on the open interval $(0,3)$, and $f(0) + 2f(1) + 3f(2) = 6$, $f(3) = 1$. Show that there exists $\xi \in (0,3)$, such that $f'(\xi) = 0$.

Hint: Consider the minimum and maximum of f on interval $[0,2]$, then use the intermediate value theorem to show there exists $c \in [0,2]$, such that $f(c) = 1$. To get some intuition draw a couple of examples, or start with the case where $f(x)$ is positive on the interval $[0,2]$.

8 *Old exam problem.*

a) Solve the initial value problem

$$\frac{y'(x)}{2x} - y(x) = 1, \quad y(1) = 2.$$

b) Show that the solution is uniformly continuous on $[1, 2]$ but not on $(1, \infty)$.