Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 12
Deadline: Nov. 28

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

You can pose questions regarding homework or lecture etc. on the discussion forum Digital Mattelab, see https://wiki.math.ntnu.no/ma1101/2021h/start.

1 Verify that $y = e^x$ and $y = e^{-x}$ are solutions of the differential equation

$$y'' - y = 0.$$

Are any of the following functions solutions?

a) $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$

b) $\cos(x)$

c) x^e

Justify your answers.

2 Solve the linear equations below.

a)

$$\frac{dy}{dx} + y = e^x$$

b)

$$\frac{dy}{dx} - \frac{2y}{x} = x^2$$

3 Solve the initial-value problem below.

$$\begin{cases} \frac{dy}{dx} + 3x^2 y = x^2 \\ y(0) = 1 \end{cases}$$

4 State the order of the given differential equations below and whether it is linear or nonlinear. If it is linear, is it homogeneous or nonhomogeneous?

a)

$$y''' + xy' = x\sin(x)$$

b)

$$\frac{d^3y}{dt^3} + t\,\frac{dy}{dt} + t^2 y = t^3$$

c)

$$\cos(x)\,\frac{dx}{dt} + x\sin(t) = 0$$

$\boxed{5}$ Solve the integral equations below.

a)

$$y(x) = 2 + \int_0^x \frac{t}{y(t)}\,dt$$

b)

$$y(x) = 3 + \int_0^x e^{-y(t)}\,dt$$

*Hint: Compute $\frac{dy}{dx}$ and use the value of $y(0)$ in **a)** and **b)** .*

$\boxed{6}$ *Old exam problem.* Consider, for $x \geq 0$, the initial value problem

$$y'(x) - 6xy(x) = 0, \quad y(0) = 1. \tag{1}$$

a) The solution $x \mapsto y(x)$ to (1) has an inverse $y \mapsto x(y)$, $y \geq 1$. Formulate the initial value problem according to (1), but for the inverse function $x = x(y)$.

b) Determine the solution y and its inverse.

$\boxed{7}$ *Old exam problem.* Let $f\colon \mathbb{R} \to \mathbb{R}$ be the function

$$x \mapsto \begin{cases} x^2\sin(\frac{1}{x}), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Show / give a mathematical argument for:

a) f is continuous and differentiable for $x \neq 0$.

b) f is continuous and differentiable at $x = 0$.

c) f' is not continuous at $x = 0$.

You do not necessarily have to use ε/δ in this problem.