



Norwegian University of Science
and Technology
Department of Mathematical
Sciences

MA1101 Basic Calculus I
Fall 2021

Exercise set 11
Deadline: Nov. 21

You may write solutions in Norwegian or English, as preferable. The most important part is *how* you arrive at an answer, not the answer itself.

You can pose questions regarding homework or lecture etc. on the discussion forum Digital Mattelab, see <https://wiki.math.ntnu.no/ma1101/2021h/start>.

1 Evaluate the limits below.

a)

$$\lim_{x \rightarrow -\infty} \frac{x^3}{e^{-x}}$$

b)

$$\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x}$$

c)

$$\lim_{x \rightarrow +\infty} \frac{[\ln(x^x)]^{\frac{1}{2}}}{e^{3x}}$$

d)

$$\lim_{x \rightarrow 0} x \left(\ln(|x|) \right)^2$$

2 Use change of variables to evaluate the integrals below.

a)

$$\int \frac{1}{(a^2 - x^2)^{\frac{3}{2}}} dx$$

Hint: Let $x = a \sin(\theta)$.

b)

$$\int \frac{2}{1 + x^2} dx$$

c)

$$\int \frac{x^2}{(1+x^2)^2} dx$$

*Hint: Let $x = \tan(\theta)$.***3** Evaluate the given integral or show that it diverges.

a)

$$\int_0^{\infty} \frac{x}{(1+2x^2)^{\frac{3}{2}}} dx$$

b)

$$\int_0^{\frac{\pi}{2}} \tan(x) dx$$

4 Given that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, evaluate

$$\int_0^{\infty} x^2 e^{-x^2} dx \quad \text{and} \quad \int_0^{\infty} x^4 e^{-x^2} dx.$$

*Hint: Note $\frac{d}{dx} \left(-\frac{1}{2} e^{-x^2} \right) = x e^{-x^2}$ and use integration by parts.***5** Find the Trapezoid Rule approximations T_4 and T_8 for the given integral

$$\int_0^{\frac{\pi}{2}} \sin(x) dx.$$

Compare your result with the actual value of the integral.

6 Find Simpson's Rule approximations S_4 and S_8 for the given integral

$$\int_0^1 e^{-x} dx.$$

Compare your result with the actual value of the integral.

7 Determine the convergence or divergence for all cases of $k > 0$ for the integral

$$\int_2^{\infty} \frac{1}{x[\ln(x)]^k} dx.$$

Hint: Start with the case $k = 1$. Use change of variables.