



MA1101 Grunnkurs i
analyse I
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Øving 8

[1] Find the antiderivatives

$$\int \cos(ax + b)dx \quad (a \neq 0), \quad \int \frac{x+1}{\sqrt{1-x^2}}dx, \quad \int \arcsin^2 x dx,$$
$$\int \cot x dx, \quad \int \frac{dx}{\sin x}$$

[2] Use partial integration to calculate

a)

$$\int_0^1 x \cos x dx,$$

b)

$$\int_0^\pi \frac{\sin x}{e^x} dx.$$

[3] Evaluate

$$\int_{-2}^{-1} \left(\frac{1}{x^2} - \frac{1}{x^3} \right) dx, \quad \int_0^e a^x dx \quad (a > 0), \quad \int_0^{\pi/2} \sin^3 x dx, \quad \int_{e^2}^{e^3} \frac{dx}{x \ln x}, \quad \int_1^{e^2} \frac{(\ln x)^3}{x} dx.$$

[4] Evaluate

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \sqrt{\frac{i}{n}}.$$

[5] Let $f : [0, 1] \rightarrow \mathbb{R}$ be continuous and non-decreasing, and P_n , $n \in \mathbb{N}$, be the partition

$$x_0 = 0, \quad x_1 = \frac{1}{n}, \quad \dots, \quad x_{n-1} = \frac{n-1}{n}, \quad x_n = 1.$$

Show that the upper and lower Darboux sums satisfy

$$U(P_n) - L(P_n) = \frac{f(1) - f(0)}{n},$$

and therefore the Riemann integral $\int_0^1 f(x) dx$ exists.

- 6** Show with the help of $\varepsilon - \delta$, that $f(x) = x^3$ is
- a) continuous at every given point $x_0 \in \mathbb{R}$,
 - b) uniformly continuous on the interval $[0, a]$ for any given $a > 0$.