1 Find the antiderivatives

$$\int \cos(ax+b)\mathrm{d}x \quad (a \neq 0), \qquad \int \frac{x+1}{\sqrt{1-x^2}}dx, \qquad \int \arcsin^2 x \,\mathrm{d}x,$$

$$\int \cot x \mathrm{d}x, \qquad \int \frac{\mathrm{d}x}{\sin x}$$

2 Use partial integration to calculate

a)

$$\int_0^1 x\cos x \,\mathrm{d}x,$$

b)

$$\int_0^\pi \frac{\sin x}{e^x}\,\mathrm{d}x.$$

3 Evaluate

$$\int_{-2}^{-1} \left(\frac{1}{x^2} - \frac{1}{x^3}\right)dx, \quad \int_0^e a^x dx \quad (a>0), \quad \int_0^{\pi/2} \sin^3 x dx, \quad \int_{e^2}^{e^3} \frac{\mathrm{d}x}{x\ln x}, \quad \int_1^{e^2} \frac{(\ln x)^3}{x}\mathrm{d}x\,.$$

4 Evaluate

$$\lim_{n\to\infty} \sum_{i=1}^n \frac{1}{n}\sqrt{\frac{i}{n}}\,.$$

5 Let $f : [0,1] \to \mathbb{R}$ be continuous and non-decreasing, and P_n, $n \in \mathbb{N}$, be the partition

$$x_0 = 0, \quad x_1 = \frac{1}{n}, \quad \ldots, \quad x_{n-1} = \frac{n-1}{n}, \quad x_n = 1.$$

Show that the upper and lower Darboux sums satisfy

$$U(P_n) - L(P_n) = \frac{f(1)-f(0)}{n},$$

and therefore the Riemann integral $\int_0^1 f(x)\,\mathrm{d}x$ exists.

6 Show with the help of $\varepsilon - \delta$, that $f(x) = x^3$ is

a) continuous at every given point $x_0 \in \mathbb{R}$,

b) uniformly continuous on the interval $[0, a]$ for any given $a > 0$.