MA6101: Eksamen H2014, oppgave 8

alternativ løsning

Anta at f, f', f'' er kontinuerlige for alle reelle tall x og at $|f''(x)| \leq 2$. Anta også at $f(0) = f'(0) = 1$. Vis at $|f'(x)| \leq 2|x| + 1$ og at $|f(2)| \leq 7$

For $x > 0$:

$$f'(x) - f'(0) = \int_0^x f''(t)\,dt \qquad \text{(korollar 8.3.4) AFT}$$

$$f'(x) = f'(0) + \int_0^x f''(t)\,dt = 1 + \int_0^x f''(t)\,dt$$

$$|f'(x)| \leq 1 + \left|\int_0^x f''(t)\,dt\right| \qquad \text{(trekantulikhet)}$$

$$\leq 1 + \int_0^x |f''(t)|\,dt \qquad \text{Fordi: } \left|\int_a^b g(x)\,dx\right| \leq \int_a^b |g(x)|\,dx$$

$$\leq \underline{1 + 2|x|}$$

Tilsvarende for $x < 0$.

$$|f(2)| = \left|f(0) + \int_0^2 f'(x)\,dx\right|$$

$$\leq |f(0)| + \left|\int_0^2 f'(x)\,dx\right|$$

$$\leq 1 + \int_0^2 |f'(x)|\,dx$$

$$\leq 1 + \int_0^2 (2x+1)\,dx$$

$$= 1 + \left[x^2 + x\right]_0^2 = \underline{7}$$

Se også: Eksamen juni 2010

høst 2010