Norwegian University of Science and Technology

Department of Mathematical Sciences

Examination paper for
MA0301 Elementary discrete mathematics

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Permitted examination support material: D: No printed or hand-written support material is allowed. A specific basic calculator is allowed.

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Date Signature

Informasjon om trykking av eksamensoppgave	
Originalen er:	
1-sidig ☐	**2-sidig** ☒
sort/hvit ☒	**farger** ☐
skal ha flervalgskjema ☐	

Exercise 1: 10 points

Exercise 2: 20 points

Exercise 3: 15 points

Exercise 4: 20 points

Exercise 5: 15 points

Exercise 6: 10 points

Exercise 7: 10 points **Total: 100 points**

Problem 1 **Logic** (10 points) Use the laws of logic

1. **(5 points)** to simplify:

$$\big(p \wedge (\neg s \vee q \vee \neg q)\big) \vee \big((s \vee t \vee \neg s) \wedge \neg q\big)$$

2. **(5 points)** to show that:

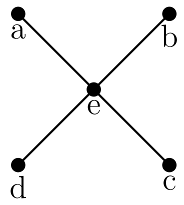
$$\big(p \to (q \vee r)\big) \Leftrightarrow \big((p \wedge \neg q) \to r\big)$$

Problem 2 **Partially ordered sets** (20 points)

1. **(3 points)** Write down the definition of a partial order.

2. **(5 points)** Does the relation $R := \{(a,a),(b,b),(c,c),(d,d),(a,b),(b,c),(c,d),(d,a)\}$ define a partial ordering on $A := \{a,b,c,d\}$?

3. **(5 points)** List the set A and express the relation R as a set of ordered pairs for the Hasse diagram



4. **(7 points)** Construct the Hasse diagram of the partially ordered set (A, R), where $A := \{a, b, c, d\}$ and $R := \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (a, c), (c, d), (a, d), (b, d)\}$.

Problem 3 **Boolean algebra** (15 points) Use Boolean algebra to

1. **(6 points)** prove that

$$x'y' + x'y + xy = x' + y$$

2. **(6 points)** prove that

$$y + x'z + xy' = x + y + z$$

3. **(3 points)** simplify

$$xyz + xyz' + x'y$$

Problem 4 **Induction** (20 points)

1. **(4 points)** Prove that

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

2. Define $\bigcup_{i=1}^{n} A_i$ by $\bigcup_{i=1}^{1} A_i = A_1$ and $\bigcup_{i=1}^{n+1} A_i = \left(\bigcup_{i=1}^{n} A_i\right) \cup A_{n+1}$. Define $\bigcap_{i=1}^{n} A_i$ by $\bigcap_{i=1}^{1} A_i = A_1$ and $\bigcap_{i=1}^{n+1} A_i = \left(\bigcap_{i=1}^{n} A_i\right) \cap A_{n+1}$.

   i) **(4 points)** Prove that

   $$\left(\bigcup_{i=1}^{n} A_i\right) \cap A = \bigcup_{i=1}^{n} (A_i \cap A).$$

   ii) **(4 points)** Prove that

   $$\overline{\left(\bigcup_{i=1}^{n} A_i\right)} = \bigcap_{i=1}^{n} \overline{A_i}.$$

   Recall that $\overline{A}$ denotes the complement of A.

3. **(8 points)** Use the trigonometrical addition formulas:

   $$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

   $$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

   to prove that for n a positive integer

   $$\left(\cos(c) + i\sin(c)\right)^n = \cos(nc) + i\sin(nc).$$

   This formula is known as De Moivre's Theorem. Note that i is the so-called *imaginary unit*, and you should use that $i^2 = -1$.

Problem 5 Binomial coefficients (15 points)

1. **(2 points)** Use the binomial theorem to compute

   $$\sum_{i=0}^{27} \binom{27}{i} (-3)^{2i+1}.$$

2. **(10 points)** Use the binomial theorem to show Vandermonde's formula

   $$\binom{a+b}{r} = \sum_{k=0}^{r} \binom{a}{k} \binom{b}{r-k}, \tag{1}$$

   where a, b, r are positive integers and $r \leq \min(a, b)$.

3. **(3 points)** Use Vandermonde's formula (1) to show that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{k}.$$

Problem 6 **Finite state automata** (10 points)

1. **(7 points)** Draw the state diagram $D(M)$ of the automaton M with states $S := \{s_0, s_1, s_2\}$, accepting states $Y := \{s_0\}$, input alphabet $I := \{a, b\}$, described in the state table $T(M)$:

	ν	
	a	b
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_2	s_2

2. **(3 points)** Write a regular expression for the language accepted by M.

Problem 7 **Graphs** (10 points)

1. **(5 points)** Is there an undirected graph with 102 vertices, such that exactly 49 vertices have degree 5, and the remaining 53 vertices have degree 6?

2. **(5 points)** If a planar graph has 12 vertices, each of degree 3, how many regions and edges does the graph have?