Norwegian University of Science and Technology
Department of Mathematical Sciences

English

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MA0301 Elementary discrete mathematics

Tuesday 1 June 2010 9:00–13:00

Permitted aids: No printed or written aids permitted. Calculator HP 30s or Citizen SR-270X

Grades to be announced: 22 June 2010

In the grading each of the ten problems counts equally.

In addition to the final examination the mid-term examination counts 20% if it is advantageous to the candidate.

Unless otherwise stated, **you should demonstrate how you arrive at your answers** (e.g. by including intermediate answers or by referral to theory or examples from the reading list).

Problem 1

In how many ways is it possible to fill in a table having 2 rows and 3 columns with integers greater than or equal to 0 such that the sum of the numbers of the first row is 5 and the sum of the numbers of the second row is 6? Two different examples of such tables follow:

$$\begin{array}{ccc} 2 & 0 & 3 \\ 2 & 4 & 0 \end{array} \qquad \begin{array}{ccc} 0 & 2 & 3 \\ 4 & 2 & 0 \end{array}$$

Problem 2

Is $((p \to r) \wedge (\neg q \to p) \wedge \neg r) \to q$ a tautology?

Problem 3

Let $A_i = \{k \in \mathbb{Z} \mid 1 \leq k \leq i\}$, that is, $A_1 = \{1\}$, $A_2 = \{1, 2\}$, $A_3 = \{1, 2, 3\}$, $\ldots$. Find $\bigcup_{i=2}^{8} A_i$, $\bigcap_{i=2}^{8} A_i$, $\bigcup_{i=1}^{\infty} A_i$, and $\bigcap_{i=1}^{\infty} A_i$,

Problem 4

Show *by induction* that every set of cardinality n (that is, the set has n elements), where n is a positive integer, has 2^n different subsets.

Problem 5

Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. The function $f\colon A \to B$ is defined by $f(x) = \lfloor 5x/4 \rfloor$ for all $x \in A$ ($\lfloor y \rfloor$ is the greatest integer less than or equal to y). Is f one-to-one (injective)? Is f onto B (surjective)?

Problem 6

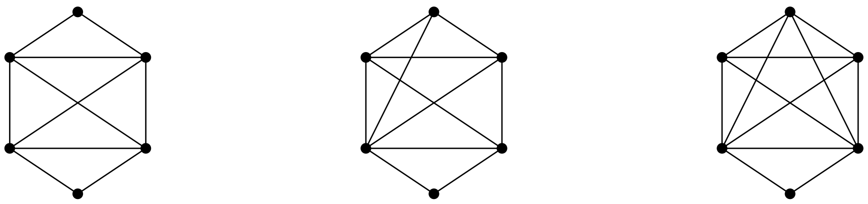
Given the language $A = \{1,\ 00\}$. Find A^n for $n = 0$, 1, 2 and 3.

Problem 7

Given the relation $\mathcal{R} = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid a = b \text{ or } a = -b\}$ on $\mathbb{Z}$. Determine whether $\mathcal{R}$ is reflexive, symmetric, antisymmetric and/or transitive. Find the equivalence classes of $\mathcal{R}$ if $\mathcal{R}$ is an equivalence relation.

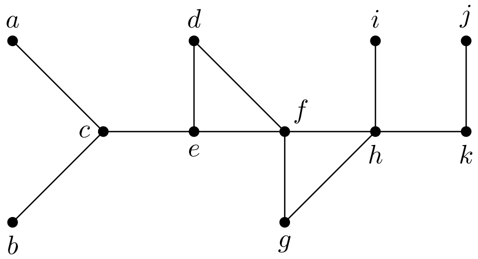
Problem 8

Determine which of the three graphs (if any) are planary (the vertices are the marked points).



Problem 9

Find a spanning tree for the graph by depth-first search. The ordering of the vertices is alphabetic. Start at vertex a. You do not have to show how you arrive at the answer, but the vertices should be labeled with the same names as below.



Problem 10

Find the shortest path from a to f and the length of it by applying Dijkstra's algorithm. You do not have to show how you arrive at the answer, but you should write down all labels at the vertices (from left to right or from top to bottom at each vertex).