Norwegian University of Science and Technology
Department of Mathematical Sciences

Contact during the exam:
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EXAM IN MA0301 ELEMENTARY DISCRETE MATHEMATICS

English
August 2012
Time: 4 hours
No printed or hand-written material is allowed during the exam.
An approved, simple calculator is allowed.

All problems have equal weight. Show your work.

Problem 1 In how many different ways can the letters in the word "kassekose" be arranged? What about "prinpripp"?

Problem 2

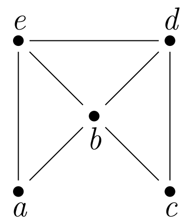
a) Show that $p \wedge \neg q \quad \Leftrightarrow \quad \neg(\neg p \vee q)$ using a truth table.

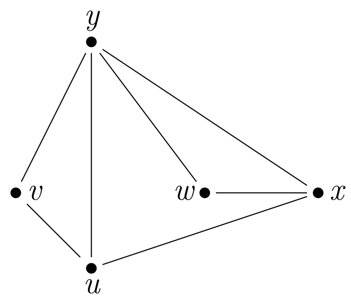
b) Decide if the statement s follows from the premises $p \leftrightarrow q$, $q \rightarrow r$, $r \vee \neg s$ and $\neg s \rightarrow q$, either by using the laws of logic and rules of inference to deduce s from the above, or by giving a counter-example.

c) Decide if the statement t follows from the premises $p \wedge q$, $p \rightarrow (r \wedge q)$, $r \rightarrow (s \vee t)$ and $\neg s$, either by using the laws of logic and rules of inference to deduce t from the above, or by giving a counter-example.

Problem 3 Use mathematical induction to show that $\sum_{i=1}^{n} i = n(n+1)/2$.

Problem 4 Are the following two graphs isomorphic? Homeomorphic?





Problem 5 A undirected, weighted graph with vertices $\{a, b, c, f, g, h, i\}$ has weighted edges as given by the following table:

vertices		weight	vertices		weight
a	g	10	a	h	17
a	b	14	b	c	9
b	f	10	b	g	3
c	f	2	f	i	7
h	i	1	g	i	4
h	g	6			

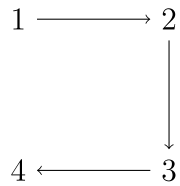
(For example, there is an edge between a and g with weight 10.)

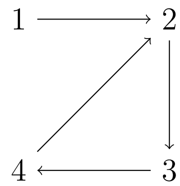
Draw the graph and use Dijkstra's algorithm to find the shortest path from a to all the other vertices in the graph.

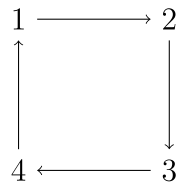
Problem 6 Let R be a relationi on a set A. The transitive closure R^+ of R is the relation

$$R^+ = R \cup R^2 \cup R^3 \cup R^4 \cup \ldots$$

a) Given the following three relations on $\{1, 2, 3, 4\}$ (given as graphs), find the transitive closure of each relation.







b) Show that if a relation is transitive, then the transitive closure of the relation equals the relation.

c) Let A and B be sets and let $f : A \to B$ be a function. We can define a relation R on A using the rule: xRy if and only if $f(x) = f(y)$.

Find the transitive closure of R.

Hint: You can use the fact that R is an equivalence relation without proof. You can use the result from the previous task even if you have not answered that task.