MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2022

Set 5

Deadline: Deadline: Monday 21.02.2022, 11:59 pm

Exercise 1. *Define the following relation on the set* $\mathbb{N}$ *of natural numbers.*

$$R = \{(m, n) \in \mathbb{N} \times \mathbb{N} | \exists l \in \mathbb{N} : n = lm\}$$

Show that R *defines a partial order.*

Exercise 2. *Consider:*

1. *The relation* $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} | x = y + 2\}$.
2. *The relation* $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x = \sin(y)\}$.
3. *For* $p \in \mathbb{Z}$ *the relation* $R_p \subseteq \mathbb{Z} \times \mathbb{Z}$ *from exercise 3 of set-4.*
4. *For* $p \in \mathbb{Z}$, *denote by* $\mathbb{Z}_p$ *the set of equivalence classes of the relation* R_p *from exercise 3 of set-4. Define the relation* $f_p = \{\langle x, y\rangle | x \in y\} \subseteq \mathbb{Z} \times \mathbb{Z}_p$.

a) *Which of the above relations defines a function? Justify your answer. (Hint: In 3. the answer might be different for different choices of* p*).*

b) *For those relations that define functions decide whether they are injective, surjective or bijective.*

Exercise 3. *Let* X *be a set and* $R \subseteq X \times X$ *an equivalence relation. Denote by* (X/R) *the set of equivalence classes of* R. *Define the relation*

$$f_R = \{\langle x, y\rangle \in X \times (X/R) | x \in y\}$$

a) *Show that* f_R *defines a surjective function. If you manage to do so you may use this fact for Exercise 2.*

b) *Define an injective function* $f_R^{-1} \subseteq (X/R) \times X$, *that has* f_R *as an inverse.*

Exercise 4. *Let* $X = \{(i, j) : i, j \in \{1, \ldots, 8\}\}$ *denote the squares of a chess board. Define the relation* $R \subseteq X \times X$, *where* $(x, y) \in R$, *if and only if a knight on the square* a, *on an otherwise empty chessboard, can reach the square* b *with finitely many moves.*

a) *Show that* $((1, 1), (3, 1)) \in R$.

b) *Show that* R *is an equivalence relation.*

c) *Determine the equivalence classes of* R.

Exercise 5. *Lewis, Zax: Exercise 6.1*

Exercise 6. *Lewis, Zax: Exercise 6.2*

Date: February 11, 2022.