MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2021

Set 5

Deadline: Wednesday 3 March, 2021, 23:59.

Exercise 1. *Show that for all integers* $m > 0$

$$\sum_{j=1}^{m} \frac{1}{j(j+2)} = \frac{m(3m+5)}{4(m+1)(m+2)}.$$

Exercise 2. *Show that for all non-negative integers* m*, we have that 3 divides the number* $a_m = 2^{2m+1} + 1$.

Exercise 3. *Let* $\{L_n\}$, $n \geq 0$ *be the sequence of Lucas numbers, which are defined recursively, i.e., for* $n > 1$, $L_n = L_{n-1} + L_{n-2}$, $L_0 = 2$, $L_1 = 1$. *Show that for positive integers* m

$$\sum_{i=1}^{m} iL_i = mL_{m+2} - L_{m+3} + 4.$$

Exercise 4. *Show that for all integers* $m > 0$

$$\sum_{i=1}^{m} (-1)^{i+1} i^2 = (-1)^{m+1} \sum_{i=1}^{m} i.$$

Exercise 5. *For each of the following relations, determine whether the relation is reflexive, symmetric, antisymmetric, or transitive:*

(1) $R \subseteq \mathbb{N} \times \mathbb{N}$ *where* aRb *if* a *divides* b,

(2) *For given a universe* $\mathcal{U}$ *and a fixed subset* C *of* $\mathcal{U}$*, define* R *on* $\mathcal{P}(\mathcal{U})$ *as follows: For* $A, B \subseteq \mathcal{U}$ *we have* ARB *if* $A \cap C = B \cap C$.

(3) *On the set* A *of all lines in* R^2*, define the relation* R *for two lines* $l_!$ *and* l_2 *by* $l_1 R l_2$ *if* l_1 *is perpendicular to* l_2.

(4) R *is the relation on* $\mathbb{Z}$ *where* xRy *if* $x + y$ *is odd.*

Exercise 6. *Define* $R \subseteq \mathbb{N}^2 \times \mathbb{N}^2$ *the relation* $((a,b),(c,d)) \in R \Leftrightarrow ad = bc$. *Show that* R *is an equivalence relation.*

Exercise 7. *Let* $A = \{1,2,3,4,5\} \times \{1,2,3,4,5\}$. *Define the relation* R *on* A *by* $(x,y)R(u,v)$ *if* $x+y = u+v$. *Show that* R *is an equivalence relation on* A *and determine the equivalence classes* $[(1,3)]$, $[(2,4)]$ *and* $[(1,1)]$.

Exercise 8. *If* $A = \{1,2,3,4,5,6,7\}$*, define the relation* R *on* A *by* xRy *if* $x - y$ *is multiple of 3. Show that* R *is an equivalence relation on* A *and determine the equivalence classes and partition of* A *induced by* R.

Date: Deadline: Wednesday 3 March, 2021, 23:59.