

MA0301  
ELEMENTARY DISCRETE MATHEMATICS  
NTNU, SPRING 2021

SET 9

**Deadline: Wednesday 7th April, 2021, 23:59.**

**Exercise 1.** Use the laws of logic to show that  $(p \rightarrow (q \vee r)) \iff ((p \wedge \neg q) \rightarrow r)$ .

**Exercise 2.** Does the relation  $R := \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (c, d), (d, a)\}$  define a partial ordering on  $A := \{a, b, c, d\}$ ?

**Exercise 3.** Use boolean algebra to:

a. Simplify  $xyz + xyz' + x'y$ .

b. Prove that  $y + x'z + xy' = x + y + z$ .

**Exercise 4.** Use induction to prove that  $\sum_{n=1}^k \frac{1}{(2n-1)(2n+1)} = \frac{k}{2k+1}$ .

**Exercise 5.** Define the function  $f(x) := 2x - 3$  from  $\mathbb{R}$  to  $\mathbb{R}$ . Show that  $F$  is surjective and injective. Find its inverse function  $f^{-1}$ .

**Exercise 6.** Let  $X, Y$ , and  $Z$  be sets. Prove that  $\overline{(X \cap Y \cap Z)} = \overline{X} \cup \overline{Y} \cup \overline{Z}$ .

**Exercise 7.** A certain ice cream store has 31 flavors of ice cream available. In how many ways can we order a dozen ice cream cones if:

a. we do not want the same flavor more than once?

b. a flavor may be ordered as many as 12 times?

**Exercise 8.** Determine the number of integer solutions for  $x_1 + x_2 + x_3 + x_4 + x_5 < 40$  where,

a.  $x_i \geq 0, 1 \leq i \leq 5$ .

b.  $x_i \geq -3, 1 \leq i \leq 5$ .

**Exercise 9.** Let  $S$  be a finite set with  $|S| = N$  and let  $c_1, c_2, c_3, c_4$  be four conditions, each of which may be satisfied with one or more of the elements of  $S$ . Prove that  $N(\bar{c}_2\bar{c}_3\bar{c}_4) = N(c_1\bar{c}_2\bar{c}_3\bar{c}_4) + N(\bar{c}_1\bar{c}_2\bar{c}_3\bar{c}_4)$ .

**Exercise 10.** Determine the number of positive integers  $n$ ,  $1 \leq n \leq 2000$  that are not divisible by 2, 3, 5, but are divisible by 7.